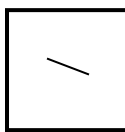



Folding to Construct Shapes

Use a pencil, straightedge (if you use a ruler as your straightedge do not measure!), and patty paper to complete the constructions described in the problems below. You can fold the patty paper to create creases and to place segments or angles on top of each other.

- 1) Draw a line segment on the patty paper, making sure the line segment is not parallel to the edges of the paper.



- a) Construct a line segment that is perpendicular to your original segment. A perpendicular line segment should form 90-degree angles with your original segment at the point of intersection.


i) Use pictures (a fold could be represented by a dotted line, ) and words to describe how you constructed the perpendicular line segment.

ii) Your classmate Jared doesn't think your new segment is perpendicular to your original line segment. Write a convincing mathematical explanation that would convince him.

- b) Now construct a line segment that is a perpendicular bisector to your original segment.

A perpendicular bisector should

- form 90-degree angles with your original segment at the point of intersection, and
- intersect your original segment at its midpoint.

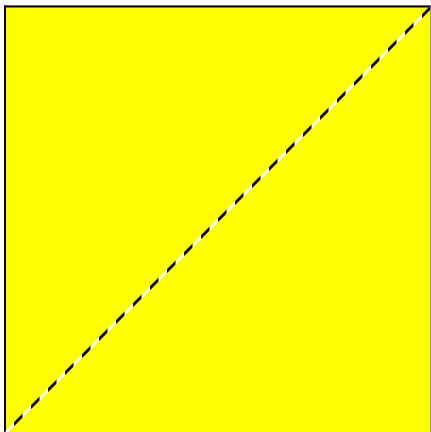
i) Use pictures (a fold could be represented by a dotted line, ) and words to describe how you constructed the perpendicular bisector.

- ii) Your classmate Jared doesn't think your new segment is a perpendicular bisector. Write a convincing mathematical explanation that would convince him.
- c) Using the same piece of patty paper, now construct a line segment that is parallel to your original segment.
- i) Use What does it mean for one segment to be parallel to another?
- ii) Use pictures and words to describe how you constructed the parallel line segment.
- iii) Again, Jared needs convincing. Write a convincing mathematical explanation that would convince him that the new segment is parallel to your original. How do you know the segment you've constructed has the properties you listed in part i)?
- 2) For each of the following, start with a freshly drawn segment on a clean piece of patty paper, making sure the line segment is not parallel to the edges of the paper. Then construct the shape.
- a) a **non-square rectangle**, with your segment as one of its sides
- b) a **square**, with your segment as one of the sides
- What are the properties of a square?
 - Use pictures and words to describe how you constructed the square.

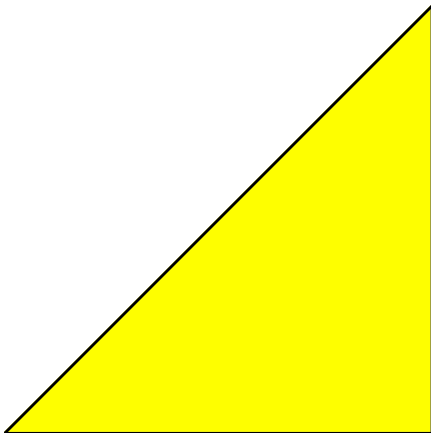
- How do you know your shape has each of the properties you listed for a square? Write a convincing mathematical explanation.
 - Before you tried your method, why did you think it would work?
 - Were there methods you tried that didn't work? If so, what were those methods?
- c) an **isosceles triangle**, with your segment as one of the two equal sides
- d) an **isosceles triangle**, with your segment as the base
- e) an **equilateral triangle**, with your segment as one of its sides
- f) choose one of the triangles you just constructed and answer the following
- What are the properties of the triangle you chose?
 - Use pictures and words to describe how you constructed the triangle.
 - How do you know your shape has each of the properties you listed for that type of triangle? Write a convincing mathematical explanation.

Warm-up for Folding to Construct Shapes

1. If you fold a square piece of paper along one of its diagonals, like this:



then you get a figure like this:



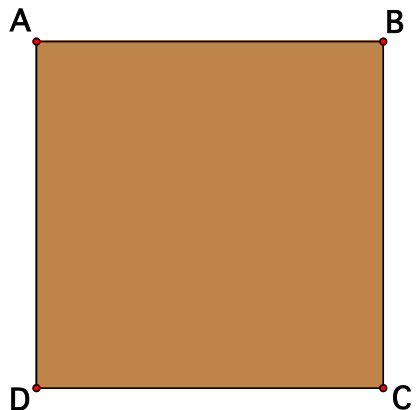
What kind of triangle is this?
Is it a right triangle? *How do you know?*

Is it an isosceles triangle? *How do you know?*

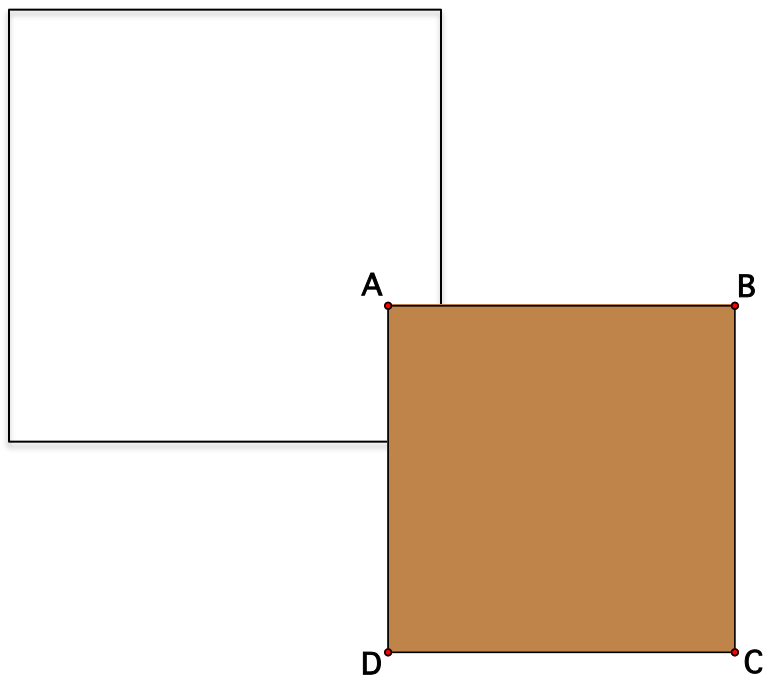
2. Where in the non-square rectangle below could you fold in order to locate the center of the rectangle? Show with dotted lines. What would you write to convince someone that you have located the center of the rectangle?



3. Show where you would fold Square ABCD in order to create a line segment parallel to segment \overline{AB}



Now show where you would fold Square ABCD in order to create a line segment perpendicular to segment \overline{AB}



Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.

Starters

(Question 1a)

To create a line segment perpendicular to the original line segment, I folded the paper

I know my new line segment is perpendicular to the original line segment because

(Question 2)

I know that the figure I created by folding has all of the properties of a square because I

(after part 3)

I know that the two triangles I have created in #2 and #3 are not congruent because:

Frames

(Before making folds for any question)

I predict that if I fold _____ **then** it will create _____.

I know my new line segment is _____ to the original line segment because the two lines form a 90-degree angle.

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- line segment
- construct
- perpendicular, parallel, perpendicular or parallel line segment
- perpendicular bisector
- 90 degree angle, intersect, point of intersection, midpoint
- half
- square
- nonsquare rectangle, square, isosceles triangle, equilateral triangle
- properties
- convince, convincing mathematical explanation



Word Chart for Folding to Construct Shapes

Spanish, French, Portuguese

* = Cognate

Words and Phrases	Academic Language Version	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Construct	To create new elements with previously measured or constructed elements.	To build; to assemble. To put something together.	Constructing Constructed Constructs	Build Make Create Form To put together	*Construir *Construire *Construir	
Perpendicular	Intersecting at or forming right angles A line or plane that intersects another line or plane at a right angle	A line or plane that comes together to form a square angle, a 90-degree angle.	Perpendicularity Perpendicularly	Vertical Upright Straight up	*Perpendicular *Perpendiculaire *Perpendicular	



Words and Phrases	Academic Language Version	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Parallel	Designating two or more coplanar lines that do not intersect	Having the same tendency or direction. Being an equal distance apart everywhere. Never intersecting.	Paralleled Paralleling Parallels	Parallelogram Equidistant In the same direction	*Paralelo(a) *Parallèle *Paralelo(a)	
Angles	Shape formed by two straight lines or surfaces that meet	a corner; a shape formed by two lines meeting at a point.	Angle Angled Angling Angular	Corner	*Ángulo *Angle *Ângulo	
Intersection	A point or locus of points where one line, surface, or solid crosses another; set that contains elements shared by two or more sets	A junction; a place where two or more things come together	Intersecting intersect intersected	Crossroads Junction Meeting point	*Intersección *Intersection *Interseção	
Bisector	A figure (line, ray) that divides into two equal parts.	Cuts or splits into two.	Bisect Bisection Bisects Bisectional	Break in two Cut in half Split evenly	*Bisectriz *Bissectrice *Bissetriz	



Words and Phrases	Academic Language Version	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Midpoint	A point on a line segment that divides it into two equal parts The halfway point of a line segment	A point at the center or middle.	Midpoints	Center Middle Halfway point Midway	*Punto medio *Point médian Ponto central	
Isosceles Triangle	A triangle that has two equal sides. The angles opposite these sides are also equal.	A triangle with two equal sides.	---	---	*Triángulo isosceles *Triangle isocèle *Triângulo isósceles	
Equilateral Triangle	A triangle with all three sides of equal length and whose angles are all 60 degrees.	A triangle with all three sides of equal length.	---	---	*Triángulo equilátero *Triangle équilatérale *Triângulo equilátero	



Words and Phrases	Academic Language Version	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
'Patty Paper'	Waxed squares of paper used by students to discover geometric properties by tracing and folding.	Waxed squares of paper used by restaurants and butchers to separate meat.	---	Tracing paper	<p>Papel de cera (wax) Papel de hamburguesa</p> <p>Papel de calcar (tracing paper)</p> <p>Papier de cire (wax) Papier de steak haché</p> <p>Papier Calque (tracing paper)</p> <p>Papel de cera (wax) Papel de hambúrguer</p> <p>Papel Vegetal (tracing paper)</p>	
Half	<p>Either of two equal parts of a whole.</p> <p>Either of two equal parts, which together compose a value twice as large as either one of the halves.</p>	One of two equal parts; 50%;	Halves Halved	<p>Split in two</p> <p>Split down the middle</p> <p>Bisected</p> <p>Divide equally in two</p>	<p>Mitad</p> <p>Moitié</p> <p>Metade</p>	



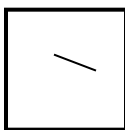
Words and Phrases	Academic Language Version	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Square	A 4-sided regular polygon with all sides equal and all internal angles 90 degrees; two-dimensional	A rectangle with all four sides equal	Squares Squarely	Equal sides Congruent sides	Cuadrado Carré Quadrado	
Line Segment	A straight line that links two points without extending beyond them.	Unbroken and bounded portion of line.	Segmented line Line segments Segmenting lines	Part of line Portion of line Section of line	*Segmento de línea *Segment *Segmento de linha	

I. “Form” vs. “Construct”

The teacher may wish to consider using the word “form” instead of “construct” as it appears throughout the problem (e.g., Construct a line segment that is perpendicular to your original segment). As in English, the word construct (i.e., construire, construire) evokes notions of erecting physical structures. “Form,” on the other hand, may help more clearly convey to the student that s/he is to manipulate, change the appearance of the sheet of paper.

II. “Line” vs. “Line Segment”

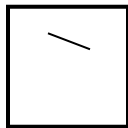
The trace below is a “line segment,” and is described as such in the problem. Be aware, however, that students might view the trace below as a “line,” and therefore become misled to think that “line *segment*” is something other than the image depicted. It may help to clarify the difference and similarity between the two words before beginning the problem (i.e., a line segment is an unbroken and bounded portion of a line; a line is infinite).



Doblando Para Construir Formas

Usar un lapic, una regla (si se usa una regla, no mires las líneas de medida) y “*patty paper*” para completar las construcciones que se describen en los siguientes problemas. Podrás doblar el “*patty paper*” para crear pliegues y poner segmentos lineales o ángulos encima uno de otro.

1. Dibujar un segmento lineal en el *patty paper*, asegurándote de que dicho segmento no sea paralelo al borde del papel.



- a. Construir un segmento lineal perpendicular al segmento original. El segmento perpendicular formará ángulos de 90 grados con el segmento original, en el punto de intersección.
 - i. Dibujar (el papel doblando una vez puede representarse con una línea de puntos) y describir como construiste el segmento lineal perpendicular.
 - ii. Tu compañero Jared no cree que tu nuevo segmento es perpendicular al segmento original. Dar una explicación matemática de tu trabajo que ayude a convencerlo.
- b. Ahora construir un segmento lineal que sea un bisector perpendicular a tu segmento original.

Un bisector perpendicular tiene que

- formar ángulos de 90 grados con tu segmento original en el punto de intersección, e
- intersectar tu segmento original a su punto medio

- i. Dibujar (el papel doblado una vez puede representarse con una línea de puntos) y describir como construiste el bisector perpendicular.
 - ii. Tu compañero Jared no cree que tu nuevo segmento es un bisector perpendicular. Dar una explicación matemática de tu trabajo que ayude a convencerlo.
 - c. Usando el mismo *patty paper*, construir ahora un segmento lineal que sea paralelo a tu segmento original.
 - i. Qué quiere decir que un segmento es paralelo a otro segmento?
 - ii. Dibujar y describir cómo construiste un segmento lineal paralelo.
 - iii. Otra vez Jared necesita que lo convenzas. Escribir una explicación matemática de tu trabajo que ayude a convencerlo. Cómo sabes que el segmento que has construido tiene las propiedades que indicas en el apartado i?
2. Para cada uno de los siguientes apartados a)-f), empieza con un dibujo de un segmento nuevo en una hoja de papel diferente, asegurándote de que dicho segmento lineal no sea paralelo a la orilla del papel. Entonces construye la forma.
- a. Un **rectángulo que no sea cuadrado**, con tu segmento como uno de sus lados
 - b. Un **cuadrado**, con tu segmento como uno de sus lados



- Cuáles son las propiedades de un cuadrado?
 - Dibujar y describir como construiste el cuadrado.
 - Cómo sabes que el cuadrado tiene cada una de las propiedades que tu describiste? Escribir una explicación matemática convincente.
 - Antes de comprobar tu método, porqué pensaste que era el método correcto?
 - Tratastes algunos métodos que no eran correctos? Cualés eran esos métodos?
- c) un **triángulo isosceles**, con tu segmento como uno de los dos lados iguales
- d) un **triángulo isosceles**, con tu segmento como la base
- e) un **triángulo equilateral**, con tu segmento como uno de sus lados
- f) escoger uno de los triángulos que has construído y contesta lo siguiente:
- Cuáles son las propiedades del triángulo que tu escogistes?

