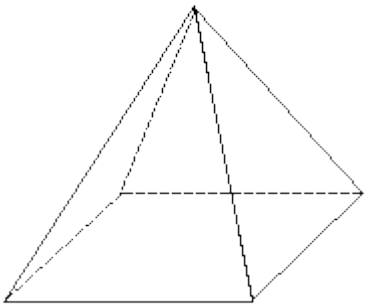
Constructing in 3 Dimensions

1. A *pyramid* is a polyhedron with a face called its base, which can be any polygon, and its other faces are all triangles meeting at a single point off of the base. Here are two pyramids, one with a pentagon as base and the other with a rectangle as base:

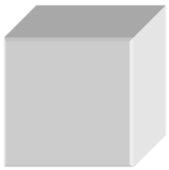


   a. Using your marshmallows and toothpicks, construct pyramids such that the triangular faces have sides which are one toothpick long. Start with a pyramid having a three-sided (triangular) base, then construct one with a four-sided base, then five, etc., until you can no longer construct more pyramids. For each pyramid you construct, record all information in the attached table.

   b. Eventually you were unable to make more pyramids. Why was this case?

   c. Look at your table. Do you notice any patterns related to the number of faces on a pyramid, the number of marshmallows needed to construct that pyramid, and the number of toothpicks needed? Describe as many relationships as you can.

2. There are many other polyhedrons besides pyramids. One well-known polyhedron is the cube, which has 6 square faces:



   a. Construct a cube with your marshmallows and toothpicks. Record the number of faces, number of marshmallows used, and number of toothpicks used under the heading of "Cube and Other Polyhedron" in the table.

   b. How do the relationships between number of faces, marshmallows, and toothpicks for the cube compare to the relationships you listed in part 1. c.?

Task 13:
Constructing in 3 Dimensions

Extension Question:

Create another 6-sided polyhedron which is neither a cube nor a pyramid. Sketch the polyhedron below and/or describe it clearly with words. Did the number of marshmallows and toothpicks used fall in line with your earlier work? Can you describe a general rule which relates these attributes for all polyhedrons? You may wish to construct other polyhedrons and record their attributes in the table before you describe a pattern.

Pyramids (Problem 1)			
Number of Sides of the Base	**Number of Faces of the Solid Figure**	**Number of Marshmallows Used**	**Number of Toothpicks Used**
Cube and Other Polyhedron (Problem 2 and Extension)			

Warm-up for Constructing in Three Dimensions

1. A pyramid is made of a *base* and *faces*. Using marshmallows and toothpicks, construct a pyramid. After you are finished, answer questions **1*a*** and ***1b*** below.

   1a. How many faces does your pyramid have?

   _____

   1b. What is the shape of your pyramid's base?

   _____

2. Now turn to a partner and discuss your pyramids. Are your pyramids the same? If not, explain how they are different. Write about their similarities and differences on the lines below.

Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning.

Starters

One relationship I noticed between the number of faces on a pyramid and the number of toothpicks I needed was _____

_____

One relationship I noticed between the number of faces on a pyramid and the number of marshmallows I needed was _____

_____

Frames

For every _____ that I added to a pyramid, I needed _____

_____

The cube is a _____ with _____

_____

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- 3-dimensional; Polyhedron; Prism
- Pyramid; Cube
- Base; Faces
- Pentagon
- Construct

Word Chart for Constructing in Three Dimensions

Spanish, French, Portuguese
* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
3-Dimensional	Having depth, along with width and length.	---	---	3-D Polyhedron	*Tridimensional En trois dimensions *Tridimensional	
Polyhedron	3-dimensional object whose surface area is series of polygons.	---	Polyhedrons Polyhedral	3-D object Faces Edges	*Poliedro *Polyèdre *Poliedro	
Base	Side of a geometric figure to which an altitude can be drawn, constructed.	Bottom side or part of a structure.	Bases	Bottom Foundation Perpendicular to the altitude	*Base *Base *Base	

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Constructing in Three Dimensions

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Pentagon	Polygon with five sides.	---	Pentagons Pentagonal	Two-dimensional shape with five sides	*Pentágono *Pentagone *Pentágono	
Construct	To create new elements with previously measured or constructed elements.	To build; to assemble. To put something together.	Constructing Constructed Constructs	Build Make Create Form To put together	*Construir *Construire *Construir	
Pyramid	A polyhedron that has a base and three or more triangular faces that meet at a point above the base.	---	Pyramids	---	*Pirámide *Pyramide *Pirâmide	

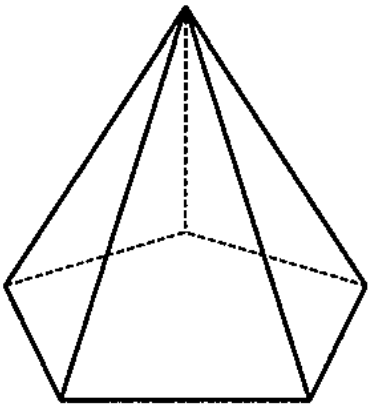
Support Notes for Constructing in Three Dimensions

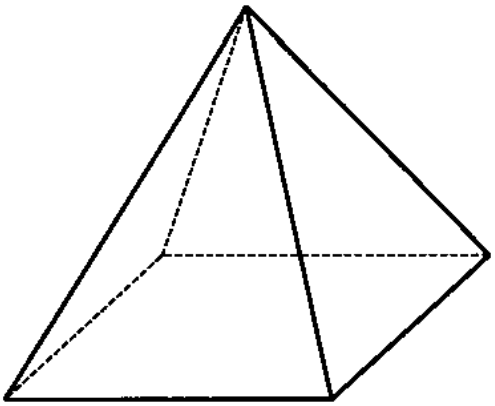
I. "Form" vs. "Construct"

The teacher may wish to consider using the word "form" instead of "construct" as it appears throughout the problem (e.g., *Construct* a **square** with exactly **¼ the area** of the original square; *Construct* a **triangle** with exactly **¼ the area** of the original square). As in English, the word construct (i.e., construir, construire) evokes notions of erecting physical structures. "Form," on the other hand, may help more clearly convey to the student that s/he is to manipulate, change the appearance of the sheet of paper.

Construyendo en Tres Dimensiones

1. Una pirámide es un poliedro que tiene dos tipos de cara. Una de las caras se llama el *base,* lo cual puede tomar la forma de cualquier polígono. Las otras caras son triángulos, las cuales convergen en un punto específico alejado del base. Abajo hay dos pirámides, uno con base pentagonal y otro con base rectangular:



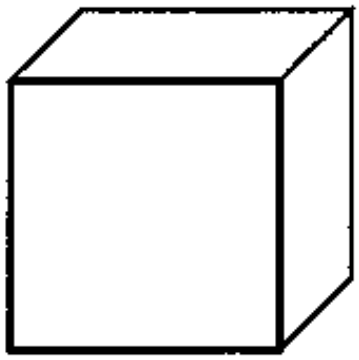


a. Usando sus malvaviscos (marshmallows) y palillos, construya pirámides que tengan caras triangulares cuales lados miden 1 palillo en longitud. Comienza con un pirámide que tenga un base de tres lados (triangular), después construya una pirámide con base de cuatros lados, después de cinco, y así sucesivamente, hasta no poder construir más pirámides. Para cada pirámide que usted construya, anote toda la información en "Table 1" de la tercera página.

b. Al final, usted no pudo construir más pirámides. Explique la razón.

c. Mire su tabla (*Table 1*). Observa algo relacionado al número de caras de una pirámide, el número de malvaviscos necesario para construir esa pirámide, y el número de palillos necesario? Describa todas las relaciones que usted pueda.

2. Además del pirámide, hay muchos otros tipos de poliedros. Uno muy conocido es el cubo, lo cual tiene seis caras cuadradas y equivalentes.



a. Construya un cubo con sus malvaviscos y palillos. Anote el número de caras, número de malvaviscos usado, y el número de palillos usado dentro de la sección titulada "Cube and Other Polyhedron (Problem 2 and Extension)" de su tabla (Table 1).

b. ¿Cómo se comparan las relaciones entre el número de caras, malvaviscos, y palillos para el cubo, y las relaciones que usted anotó para la pregunta 1c.?

Pregunta Suplementaria

Cree otro poliedro de 6 caras que no sea cubo ni pirámide. Dibuje el poliedro en el espacio abajo y/o descríbalo claramente en palabras. ¿Será que el número de malvaviscos y palillos que usted usó para este nuevo poliedro tienen algo en similar con el número de malvaviscos y palillos que usted usó para las actividades anteriores? Puede usted describir una regla que, en general, relaciona estos atributos de los poliedros? **Pista:** se le ayudaría construir otros poliedros y anotar sus atributos en la tabla (table 1) antes de describir una regla.

Table 1

Pirámides (Problema 1)

Número de lados del base	Número de caras del poliedro	Número de malvaviscos usados	Número de palillos usados

Cubo y Otros Poliedros (Problema 2 y Suplementaria)
