Investigating Area by Folding Paper

For each problem, start with a square sheet of paper and make folds in the sheet of paper to construct a new shape, then explain how you know the shape you constructed has the specified area.

1. Construct a **square** with exactly **¼ the area** of the original square.

   Explain how you know it has ¼ the area:

2. Construct a **triangle** with exactly **¼ the area** of the original square.

   Explain how you know it has ¼ the area:

3. Construct **another triangle**, also with **¼ the area**, which is **not congruent** to the first one you constructed.

   Explain how you know it has ¼ the area:

4. Construct a **square** with exactly **½ the area** of the original square.

   Explain how you know it has ½ the area:

5. Construct **another square**, also with **½ the area**, which is oriented differently than the one you constructed in #4.

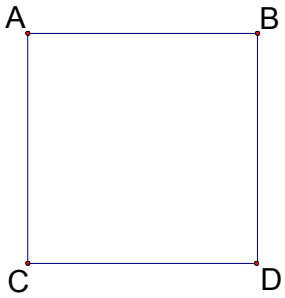
   Explain how you know it has ½ the area:

Convincing Mathematical Explanations

Convincing mathematical explanations stand up to any challenge and can convince others of a mathematical result. Two conditions must be satisfied:

- **Convincing explanations use facts, not opinions, to support claims.**

The figure below is a square.

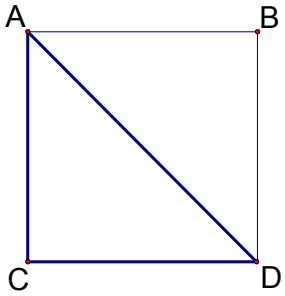


What is the measure of ∠ACD?

Opinion: ∠ACD is 90 degrees because it *looks* like an L.
Fact: ∠ACD is 90 degrees because it is one of the 4 angles in a square and all 4 angles of a square are 90 degrees.

- **Convincing explanations are complete and don't leave any gaps or holes.**

The figure below is a square.



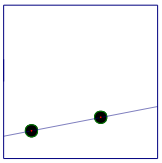
What is the name of the figure in bold?

Explanation with gaps: This figure is an isosceles right triangle. I know this because two of the triangle's sides are sides of the square, so they must be the exact same length.
Complete explanation: This figure is an isosceles right triangle. I know this because two of the triangle's sides are sides of the square, so they must be the exact same length. And ∠ACD is a right angle because it is one of the 4 angles of the square, and all angles in a square are 90 degrees.

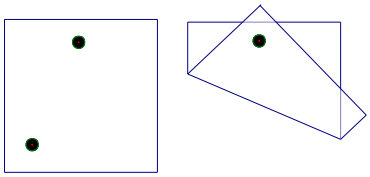
An Introduction to Patty Paper

Patty paper can be used as a construction tool in geometry. You can use it to construct a variety of geometric figures. If you're not familiar with patty paper, work through the following simple exercises and you will start to see how patty paper can support your geometric thinking.[1]

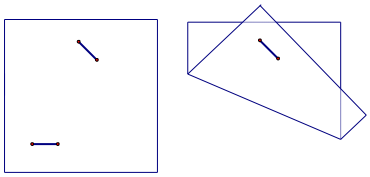
1. When patty paper is folded a crease, or line, appears. Try folding a piece of patty paper to make a line.
2. Patty paper can be used to construct a line connecting two points. Draw 2 points anywhere on a sheet of patty paper. Now fold your paper so that the crease, or line, passes through both points.



3. You can place one point on top of another using patty paper. Draw 2 points anywhere on a sheet of patty paper and try it.

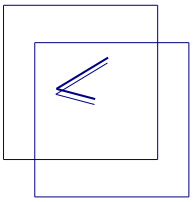


4. Like the points, you can place one line on top of another with patty paper. Draw 2 line segments on a sheet of patty paper and try it.



Once you have folded so that one segment is placed on the other, you can check to see if two segments are congruent. Are your two segments congruent? How do you know?

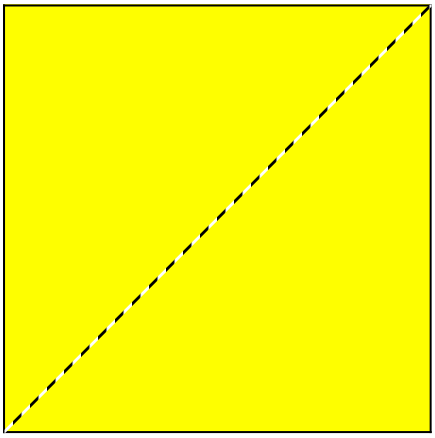
5. You can create congruent angles with patty paper. Draw an angle on a piece of patty paper. Now place a second piece of patty paper over your first and trace your angle onto this second sheet.



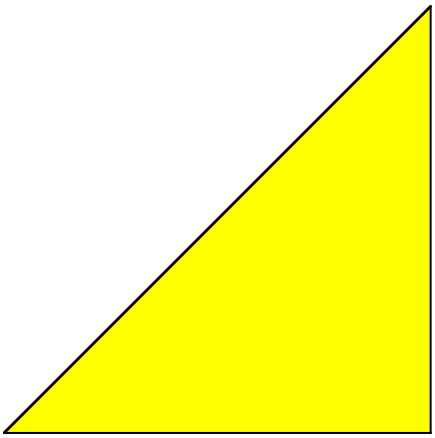
[1] This is an adaptation of the Introduction to Patty Paper Geometry activity that originally appeared in Michael Serra's *Patty Paper Geometry.* Emeryville, CA: Key Curriculum Press.

Warm-up for Investigating Area by Folding Paper

1. If you fold a square piece of paper along one of its diagonals, like this:

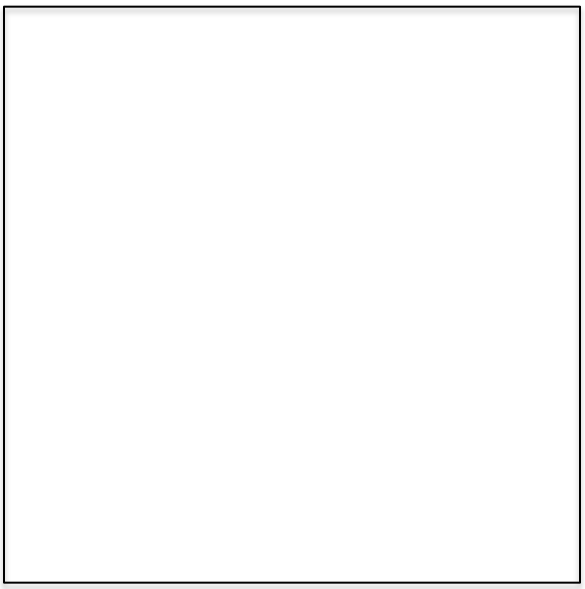


then you get a figure like this:



What kind of triangle is this? What is the relationship between the area of this triangle and the area of the original square? Explain how you decided on the relationship.

2. Where in the square below could you fold in order to construct a rectangle with exactly ½ the area of the square? How do you know your rectangle has exactly ½ the area of the square?



Sentence Starters & Frames for Investigating Area by Folding Paper

Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how it will support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.

Starters

(part 1 and 2)

To create a square with ¼ the area of the original square piece of patty paper, I folded:

_____

To create a triangle with ¼ the area of the original square piece of patty paper, I folded:

_____

(after part 2)

The way I folded the paper to create a square with ¼ the area and a triangle with ¼ the area of the original square piece of patty paper was different because:

_____

(after part 3)

I know that the two triangles I have created in #2 and #3 are not congruent because:

Frames

(part 1 and 2)

To create a _____with_____ the area of the original square piece of patty paper, I folded:

_____

(after part 2)

The way I folded the paper to create a _____ with ¼ the area and a triangle with ¼ the area of the original piece of patty paper was different because:

_____

(after part 3)

I know that the two triangles I have created in #2 and #3 are not _____because they cannot be transformed or manipulated in a way that allows me to place one exactly over the other.

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- Congruent
- Area
- Triangle; square
- Construct a square; construct a triangle; square or triangle that you constructed
- Side lengths are equal
- Oriented differently
- Base
- Height
- Reflection
- Fold
- Specified

Word Chart for Investigating Area by Folding Paper

Spanish, French, Portuguese
* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Construct	To create new elements with previously measured or constructed elements.	To build; to assemble. To put something together.	Constructing Constructed Constructs	Build Make Create Form To put together	• Construir* • Construire* • Construir*	
Congruent	Two geometric figures in the plane are congruent if one can be obtained from the other by some combination of rotations, reflections, and translations	The same or equal in some way; same size and shape.	Congruently Congruence	Congruous Same shape	• Congruente* • Congruent(e)* • Coincidente	

Word Chart for Investigating Area by Folding Paper

Words and Phrases	Academic Language Meaning	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Oriented Differently	Orientation: Position in relation to a particular point. Oriented differently: in a different position relative to the same point.	Location with respect to points on a compass. Positioned in a different direction or manner.	Orienting Orientation	Positioned Arranged	• Orientado diferentemente* • Orienté différemment* • Orientado diferentemente*	
Specified Area	Area: The space covered by a figure. Specified: the particular area that is indicated in the problem.	A physical location. A particular region/area/space A stated region/area/space	Specific Specify Specifies	Particular space Stated region Given space or region	• Area especificada* • Aire/Superficie spécifiée* • Área especificada *	
A Quarter (1/4)	One of four equal parts. A fourth.	A fourth of a whole. Fifteen minutes of an hour (e.g., quarter past midnight). Twenty-five of one hundred percent.	----	Fourth One-fourth A quart Quadrant Quartile	• Cuarto* • Quart* • Quarto*	

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Investigating Area by Folding Paper

Words and Phrases	Academic Language Meaning	Everyday Language Meaning	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Equal (Equivalent)	Having the same value or amount; unit of measurement.	Being the same same or identical in quantity, quality.	Equaled Equals	Same Equates Tantamount Even Identical Matching	• Igual (Equivalente)* • Égal(e) (Équivalent(e))* • Igual (Equivalente)*	
To Fold	----	To lay one part over another part.	Folded Folds	Enclose Envelop Double Over	• Doblar • Plier • Dobrar	
A Half	Either of two equal parts of a whole. Either of two equal parts, which together compose a value twice as large as either one of the halves.	One of two equal parts; 50%;	Halves Halved	Split in two Split down the middle Bisected Divide equally in two	• Mitad • Moitié • Metade	

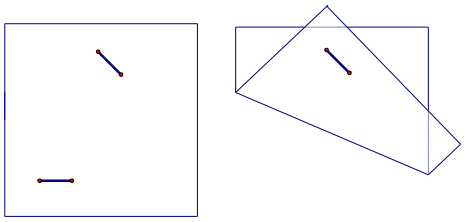
Support Notes for Investigating Area by Folding Paper

I. "Form" vs. "Construct"

The teacher may wish to consider using the word "form" instead of "construct" as it appears throughout the problem (e.g., *Construct* a **square** with exactly **¼ the area** of the original square; *Construct* a **triangle** with exactly **¼ the area** of the original square). As in English, the word construct (i.e., construir, construire) evokes notions of erecting physical structures. "Form," on the other hand, may help more clearly convey to the student that s/he is to manipulate, change the appearance of the sheet of paper.

II. "Line" vs. "Line Segment"

The traces below are "line segments," and are described as such in the problem. Be aware, however, that students might view the traces below as "lines," and therefore become misled to think that "line *segment*" is something other than the image depicted. It may help to clarify the difference and similarity between the two words before beginning the problem (i.e., a line segment is an unbroken and bounded portion of a line; a line is infinite).



Investigando el Área Doblando Papel

Para cada problema, empezar con una hoja de papel cuadrada y doblarla para construir una nueva forma; después explicar porqué la forma construída tiene el área específica.

1. Construir un **cuadrado** que tenga exactamente **¼ del área** del cuadrado original.

   Explicar porqué tiene ¼ del área:

2. Construir un **triángulo** que tenga exactamente **¼ del área** del cuadrado original.

   Explicar porqué tiene ¼ del área:

3. Construct **otro triángulo**, que tenga también **¼ del área**, sin ser **congruente** al primer triángulo construido.

   Explicar porqué tiene ¼ del área:

4. Construir un **cuadrado** que tenga exactamente **½ del área** del cuadrado original.

   Explicar porqué tiene ½ del área:

5. Construir **otro cuadrado**, que tenga también **½ del área**, pero con diferente orientación del construído en #4.

   Explicar porqué tiene ½ del área: