Finding Centers of Rotation

1. Draw a line segment $\overline{AB}$ on a piece of paper, then a point C not on $\overline{AB}$. Imagine that $\overline{AB}$ is rotated around point C, the *center of rotation*.
   a. What do you notice?

   b. Draw $\overline{AB}$ at 5 different locations during its rotation around point C.
   c. Describe the path of line segment $\overline{AB}$.

   d. Describe the path of each endpoint of $\overline{AB}$.

   e. Describe the path of the midpoint of $\overline{AB}$.

2. Take a look at the Geometer's Sketchpad file "rotate.gsp" available in the Fostering Geometric Thinking CD[1]. Explore the path the line segment takes as it rotates around the center of rotation. Click on "View several different locations…" so you can see how the segment looks at different locations as it rotates.
   a. What do you notice?

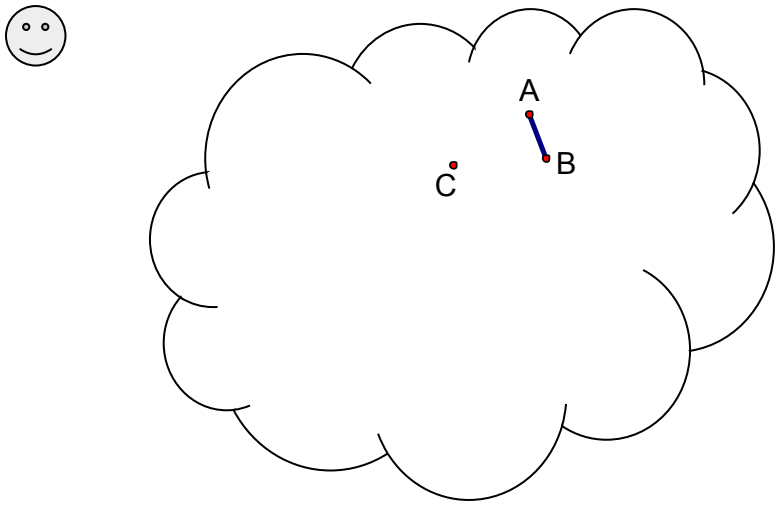
[1] If you do not have access to Geometer's Sketchpad software, you can find a web-based applet appropriate for this problem at http://www.geometric-thinking.org/rotate.htm.

b. Describe the path that each endpoint of the segment takes.

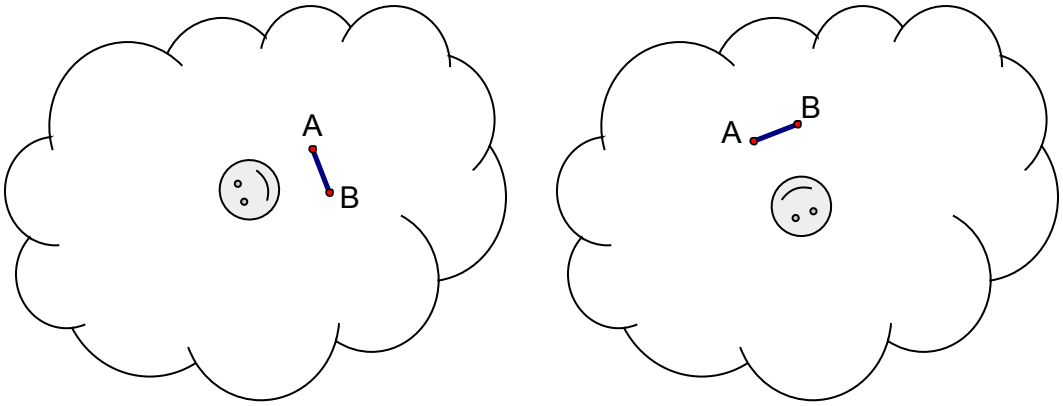
c. How did viewing the Sketchpad file affect your thinking about the 5 positions for $\overline{AB}$ from part 1?

d. Laura is confused about *centers of rotation* and her computer just crashed so she can't explore it with Geometer's Sketchpad. Keeping in mind what you learned from exploring the applet, describe what a *center of rotation* is to Laura. Make sure you tell her about things that change and the things that stay the same during a rotation.

3. Laura thinks this idea of *centers of rotation* is interesting and she wants to explore it some more. So far Laura has imagined herself watching from a distance as $\overline{AB}$ rotates around C.



She starts to wonder if she might notice anything different about rotation if she imagined herself at the center of rotation (in the system). She imagines standing on C and rotating with $\overline{AB}$ so that she is always looking at the segment as it rotates.



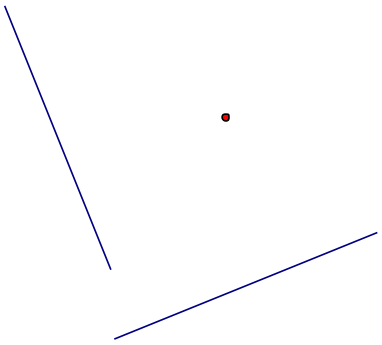
a. Imagine standing on point C and rotating so that you're always looking at $\overline{AB}$ while it rotates. What do you notice? Do you notice anything that you hadn't noticed while looking at the computer? Explain.

b. Now that you've considered centers of rotation from a few different perspectives, would you add or change anything to the description you gave Laura in part 2d?

4. Below are some pairs of congruent line segments. We know they are congruent because, for each pair, we were able to find a center of rotation and rotate the segment on the right so it fit exactly on the segment to the left.

We tried several possible points for this first segment pair and we think we've found the center of rotation. Check to see if our center is correct.

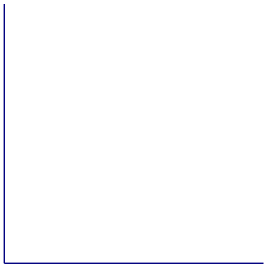
- Place a piece of patty paper over the 2 segments
- Trace the segment on the right and the center of rotation onto the patty paper
- Use your pencil to anchor the patty paper at your center of rotation and rotate the patty paper to see if the traced segment lands on top of the segment on the left



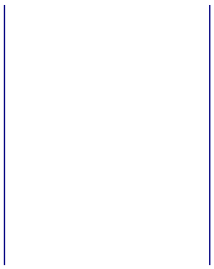
Task 7: Finding Centers of Rotation

Find the centers of rotation for the segment pairs below. Use the method outlined above to check your centers.

a) You may have tried several points, where many did not work. How did you find the center of rotation that succeeded in allowing you to rotate the right segment onto the left segment?

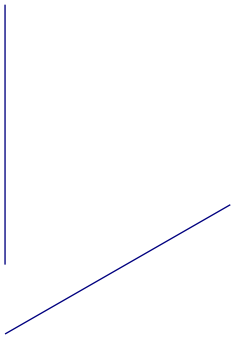


b) How did you find the center of rotation?



c) Point A is one center of rotation. (Check with your patty paper.) A rotation of 90 degrees will do it. There is another point that also serves as a center of rotation. Where is that point, and how did you find it?

d) Find 2 centers of rotation.
How did you find each center?



e) Find 2 centers of rotation.
How did you find each center?

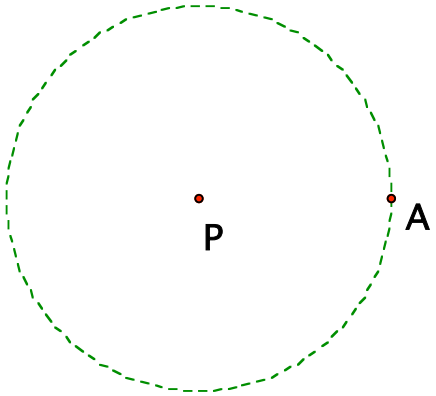
Extension:

After finding the centers of rotation for the 5 segment-pairs in part 4, Laura is pretty sure there is a general procedure that would allow her to find a center of rotation for *any* two segments. Since you were so helpful to Laura earlier, she asks for your advice again. "What procedure, or method for finding a center of rotation, will help me find a center of rotation for *any* two segments that I might draw?"

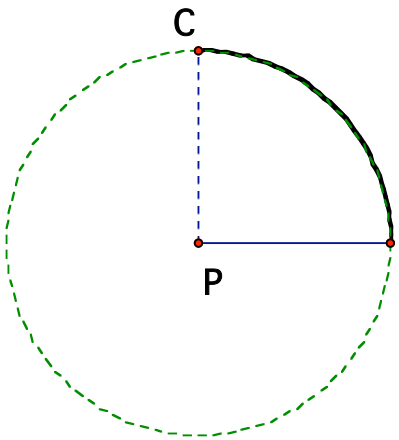
Before you respond to Laura, draw two lines segments of the same length anywhere on a piece of paper. How would you find the center of rotation P so that one of your line segments can be rotated about P, to get to the position of the other line segment, no matter where you drew those two line segments? Describe your method to Laura in a paragraph or two. Remember, your method has to work no matter where Laura decides to put her two line segments.

Warm-up for Finding Centers of Rotation

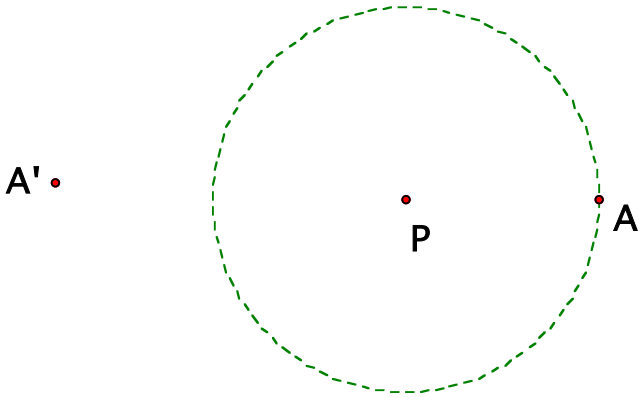
1. When a point A is rotated around center of rotation P, it moves along a circular path. The location of point A after being rotated depends on the amount of rotation.



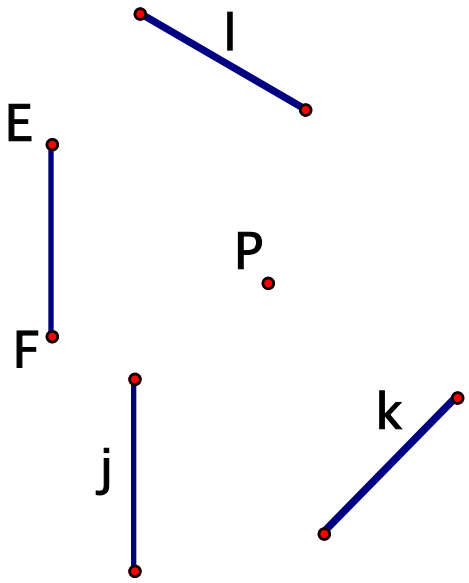
For example, if A is rotated ¼ of the circular path around P, it arrives at the location of point C in the picture below:



A new point A' is rotated 360 degrees (all the way) around center of rotation P. Draw its path of rotation:

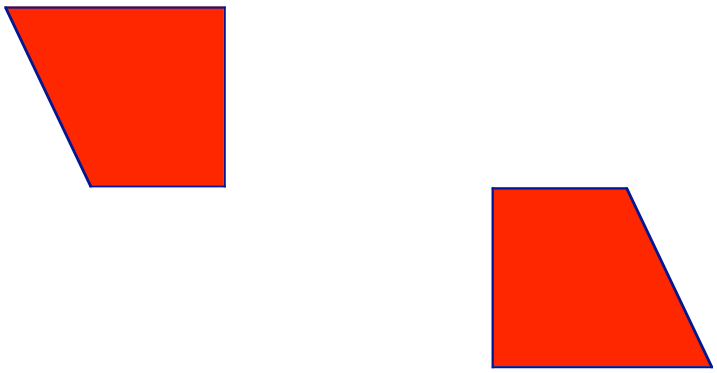


2. If line segment $\overline{EF}$ is rotated around center of rotation P, which of the three line segments, *j, k*, and *l*, could represent how $\overline{EF}$ will look as it is being rotated?



Warm-up for Finding Centers of Rotation

3. The quadrilateral on the left, below, is a rotation of the quadrilateral on the right. Find the center of rotation. Explain how you know it is the center of rotation.



Sentence Starters & Frames for Finding Centers of Rotation

Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.

Starters

To rotate the line segment AB around a center of rotation C, I _____

I noticed that when a line segment rotates around a center of rotation, the endpoints of the line segment

_____

I predict that if I rotate a triangle around a center of rotation, the vertices of the triangle will _____

_____

I know that the center of rotation for the two line segments in part 4 (a/b/c/d) must _____

_____

Frames

I noticed that when a line segment rotates around a center of rotation, the _____of the line segment remain the same distance away from the center of rotation.

I think that the center of rotation for these two line segments in part 4 is probably _____

Because _____

Sentence Starters & Frames for Finding Centers of Rotation

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- rotate; rotation; rotated; center of rotation
- line segment
- path; locations
- endpoint; midpoint
- circle
- orientation
- distance from the center of rotation is constant
- congruent
- perpendicular bisector
- point that is equidistant from both line segments
- angle of rotation

Word Chart for Finding Centers of Rotation

Spanish, French, Portuguese
* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Rotation	A transformation in which a figure is turned so that each point on the image remains the same distance from a fixed point (2-D or 3-D) or line (3-D).	Turning around; moving clockwise or counter-clockwise.	Rotations Rotating Rotated	Turn Moving in a circle	*Rotación *Rotation *Rotação	
Congruent	Two geometric figures in the plane are congruent if one can be obtained from the other by some combination of rotations, reflections, and translations	The same or equal in some way; same size and shape.	Congruently Congruence	Congruous Same shape	*Congruente *Congruent(e) Coincidente	

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Finding Centers of Rotation

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Line Segment	A straight line that links two points without extending beyond them.	Unbroken and bounded portion of line.	Segmented line Line segments Segmenting lines	Part of line Portion of line Section of line	*Segmento de línea *Segment *Segmento de linha	
Endpoint	Either of two points marking the end of a line segment.	A tip or point of termination.	Endpoints	End of line segment Limit	Punto extreme Point extrême Ponto extremo	
Midpoint	A point on a line segment that divides it into two equal parts The halfway point of a line segment	A point at the center or middle.	Midpoints	Center Middle Halfway point Midway	*Punto medio *Point médian Ponto central	
Locations	---	Particular places or positions.	Location Located Locating	Place Locale Site	*Localizaciónes/Ubicaciónes Emplacements *Localizações	

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Finding Centers of Rotation

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Path	Trajectory	Way or means to an end; designed for a particular purpose.	Paths	Pathway Passageway Way Route	Camino Chemin Caminho	
Procedure	The sequence of actions to be followed in solving a problem.	A course or mode of action.	Procedures	Process Operation Course of action	*Procedimiento *Procédure *Procedimento	
Method	A systematic way of solving a problem.	Technique or way of doing something,	Methods Methodical	Approach Technique	*Método *Méthode *Método	

Support Notes for Finding Centers of Rotation

I. *Localizaciónes vs. Ubicaciónes

While a Spanish cognate exists for "Locations" (i.e., *localizaciónes), Spanish-speaking students may not be familiar with it, as it is used rather infrequently. The term "ubicaciónes" is more commonly used to express the English meaning of "locations."

II. "Line" vs. "Line Segment"

Students are asked in this task to create and manipulate "line segments." It may help to clarify the difference and similarity between a "line" and a "line segment." That is, a line segment is an unbroken and bounded portion of a line; a line is infinite.

III. Work

"Work" is used in this task in the functional/operational sense (e.g., Bob's cell phone doesn't work). Some students may be more familiar with "work" as the verb describing an activity in which one engages to achieve a purpose or result, or "work" as the synonym for a "job" or "employment." You may find it helpful to address how "work" is used in the context of the task.

Encontrando Centros de Rotación

1. Dibuja un segmento lineal $\overline{AB}$ sobre tu papel y in punto C que no sea encima del segmento. Imagina que giras $\overline{AB}$ por el punto C, el *centro de rotación.*

    a. ¿Qué notas?

    b. Dibuja $\overline{AB}$ en 5 posiciónes diferentes mientras giras por el punto C.

    c. Describe la trayectoria del segmento $\overline{AB}$.

    d. Describe la trayectoria de cada punto final del segmento $\overline{AB}$.

    e. Describe la trayectoria del punto al medio del segmento $\overline{AB}$.

Task 7:
Encontrando Centros de Rotación

2. Mira el archivo de Geometer's sketchpad titulado "rotate.gsp" que se encuentra en el CD de Fostering Geometric Thinking. Investiga la trayectoria del segmento mientras gira por el centro de rotación. Apreta sobre "View several different locations…" para ver el segmento en posiciónes diferentes mientras gira.

   a. ¿Qué notas?

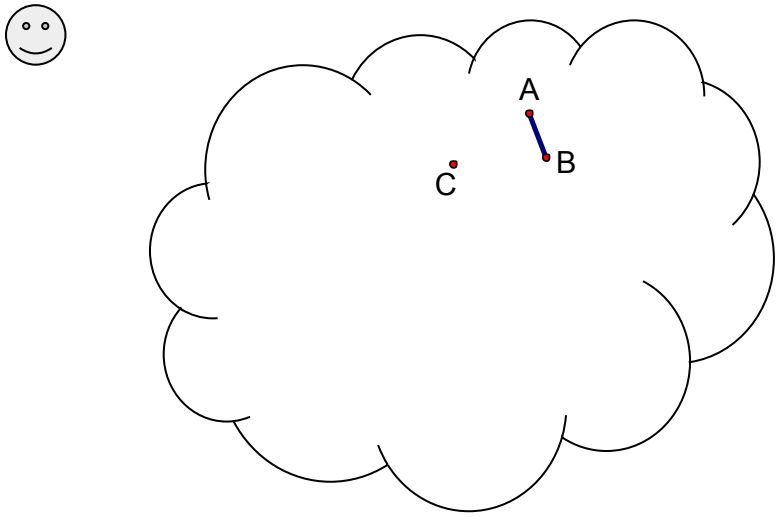
   b. Describe la trayectoria de cada punto final del segmento.

   c. ¿Cómo cambiaron tus ideas sobre las cinquo posiciónes de $\overline{AB}$?

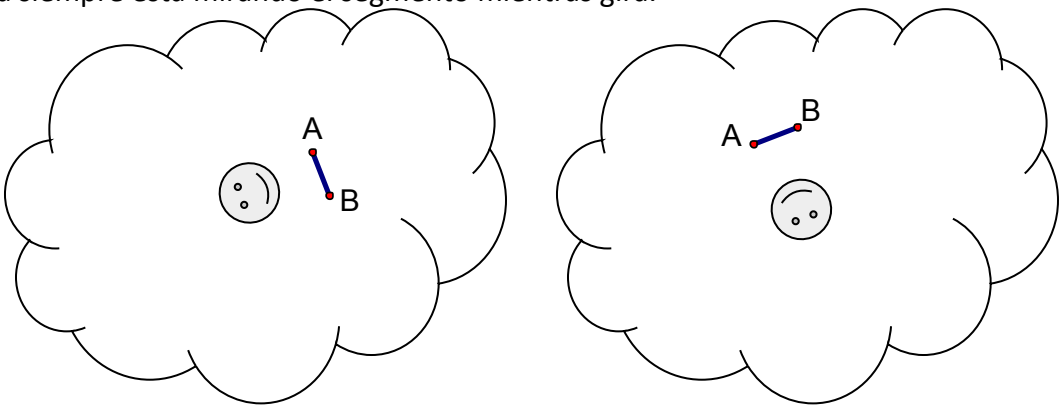
   d. Laura está un poco confundida sobre los centros de rotación y su computadora no funciona. Ayuda Laura entender lo que tu sabes de centros de rotación, acordandote de lo que aprendistes con el applet. Habla con Laura sobre las cosas que cambian y las cosas que se quedan iguales durante una rotación.

Task 7:
Encontrando Centros de Rotación

3. Laura cree que las rotaciónes son interesantes y quiere explorarlas más. Laura se ha imaginado mirando la rotación de una distancia.



Ahora Laura se pregunta si ella vería alguna cosa diferente sobre la rotación si tracta de imaginarse en el centro de la rotación. Se imagina a ella misma sobre el centro C, girando con el segmento $\overline{AB}$ de tal manera que ella siempre está mirando el segmento mientras gira.



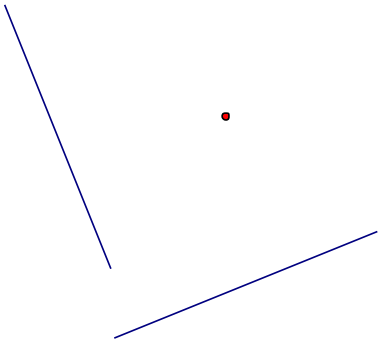
a. Imaginate a tu mismo sobre el punto C y girando de tal manera que siempre ves el segmento $\overline{AB}$ mientras gira. ¿Qué notas? ¿Ves alguna cosa que no te habías dado cuenta mientras mirabas el applet? Explica tu respuesta.

b. ¿Ahora que has considerado el centro de rotación desde diversas perspectivas, cambiarías alguna cosa de tu explicación en la parte 2d?

Task 7:
Encontrando Centros de Rotación

4. En el diagrama han dos segmentos lineales que son congruentes. Sabemos que son congruentes porque, por cada pareja, encontramos un centro de rotación y giramos el segmento a la derecha para caber exactamente sobre el segmento a la izquierda.

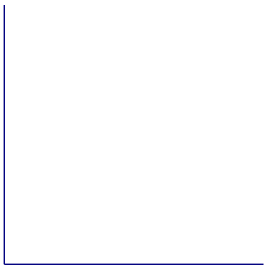
Probamos diversos puntos y creemos que hemos encontrado el centro de rotación. Mira a ver si el nuestro centro es correcto.



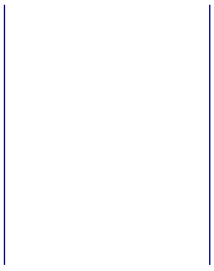
- Coloca un papel 'patty' sobre los dos segmentos
- Calca el segmento a la derecha y el centro de rotación sobre el papel 'patty'
- Usa tu lápiz para fijar el papel 'patty' en el centro de rotación, y gira el papel para ver si el segmento calcado cae sobre el segmento a la izquierda.

Busca los centros de rotación por los segmentos que siguen. Usa el método con el papel 'patty' para verificar tus centros.

a) Es posible que algunos de los puntos que probastes no funcionaron. ¿Cómo encontraste el centro de rotación que usaste para girar un segmento sobre el otro?

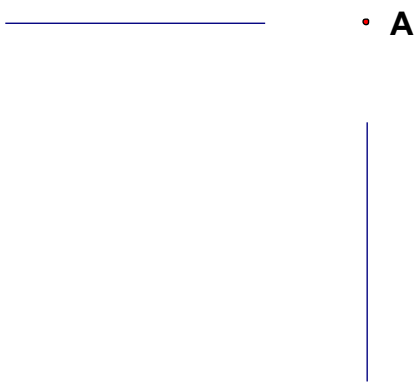


b) ¿Cómo encontraste el centro de rotación?

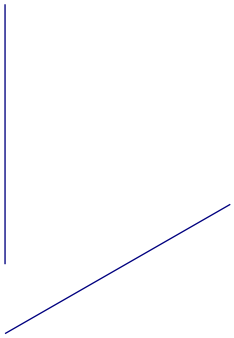


Task 7:
Encontrando Centros de Rotación

c) El punto A es un centro de rotación (Asegúrate con tu papel patty.) Una rotación de 90 grados funciona. Hay otro punto que también puede ser un centro de rotación para estos segmentos lineales. ¿Dónde está y cómo lo encontraste?



d) Busca dos centros de rotación. ¿Cómo encontraste cada uno?



e) Busca dos centros de rotación. ¿Cómo encontraste cada uno?

Extensión:

Después de encontrar los centros de rotación para las 5 parejas de segmentos en #4, Laura cree que hay un método general para encontrar centros de rotación por cualquiera de dos segmentos. Ella te pregunta otra vez por tu ayuda. "¿Qué método puedo usar para encontrar el centro de rotación por cualquiera de dos segmentos?"

Antes de responder a Laura, dibuja dos segmentos que tengan la misma medida en cualquier papel. ¿Cómo encontrarías el centro de rotación P de tal manera que uno de los dos segmentos se puede girar para caer sobre el otro, por cualquier lugar que dibujas los segmentos? Describe tu método a Laura en uno o párrafos. Recuerda que tu método tiene que funcionar por cualquier lugar dónde Laura decide dibujar sus segmentos