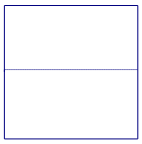
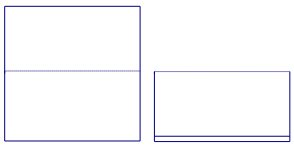
# Comparing Triangles

**Congruence: More than meets the eye.** Sometimes we think that figures are congruent because they look like exact copies, but you can't always trust your eyes. In geometry it's important to be precise so we say that a figure is congruent to another figure if we can get one by a combination of reflections (flips), translations (slides), and rotations (turns) performed on the other.

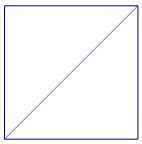
Lorna thought she had created 2 congruent rectangles within a square.



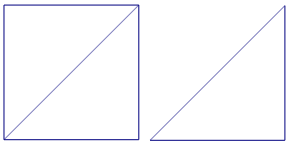
However, since she knew shapes could look like exact copies when they were not, she decided she should investigate further. She folded her paper so she could check to see if the rectangle on top could be reflected onto the one below. If the rectangles were truly congruent, one rectangle should cover the other one exactly. When she folded she found that wasn't the case. Her rectangles looked congruent but they weren't!



Next, Lorna decided she would try to create 2 congruent triangles within the square. She drew one of the diagonals of the square and exclaimed, "I have congruent isosceles right triangles!"

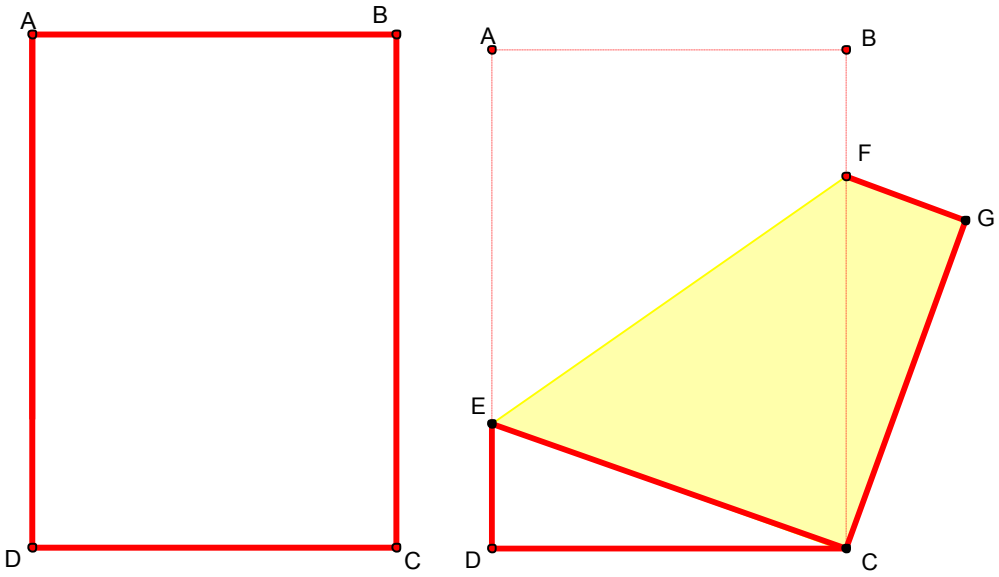


Again, she knew she better check to make sure because figures that look congruent aren't always congruent. She folded the square along the diagonal and the triangles sat exactly on top of each other – they were congruent because one was the reflection of the other!



# Task 6: Comparing Triangles

1. *Now it's your turn to compare some shapes.* Start with a piece of paper that is a non-square rectangle, and be sure it is a different size than your neighbor's rectangle. Fold your paper so that point A is directly on top of point C as shown in the picture below.




a) As in our picture above, you should see some triangles. Outline the triangles on your paper using a marker. How many did you find?

b) Compare triangles ΔEDC and ΔFGC on your paper. How are they alike?

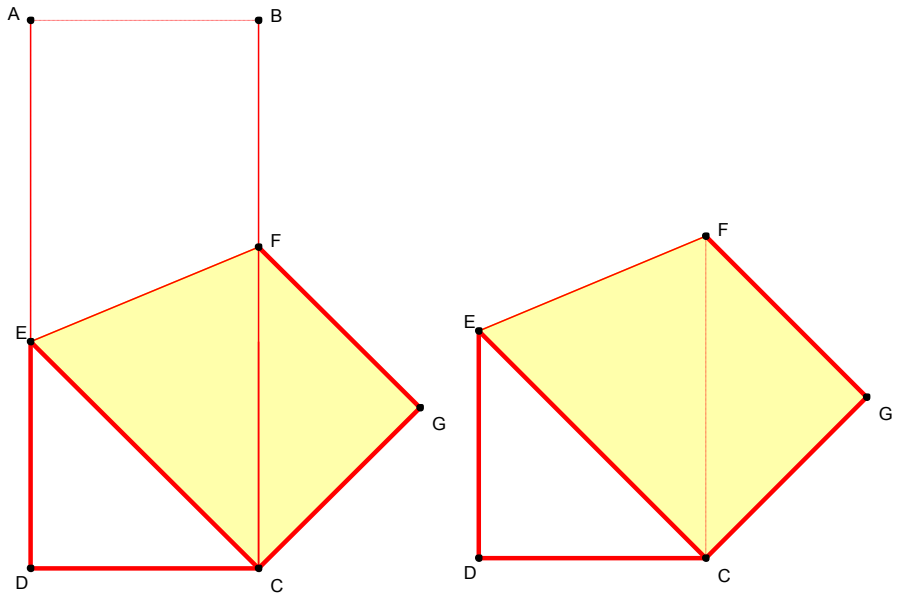
c) Your neighbor folded a different rectangle. Ask your neighbor what they noticed. What did your neighbor discover about how ΔEDC and ΔFGC are alike?

d) Julio thinks the two triangles are congruent, but cannot show it. Help Julio show congruence. Remember how Lorna thought about congruence – one triangle is congruent to another if we can get one by reflecting, translating, or rotating the other. Can you show Julio how to get ΔEDC by reflecting, translating, or rotating ΔFGC?

e) Look at ΔCEF in our picture. What is special about this triangle? Compare this triangle from your paper with your neighbor's. What do you notice?

2. *Research and advise Morgana.* Morgana is starting a small business, making and selling geometry materials. She wants your advice on a new product, isosceles folding paper.

She tells you: "I want to sell paper that creates 2 isosceles triangles that are congruent to each other. That means when you fold point A to point C, ΔEDC and ΔFGC should be isosceles triangles that are congruent to each other. Not all rectangular paper can create these isosceles triangles so I need your help. Help me find 3 different sizes of rectangular paper that will create pairs of congruent isosceles triangles."



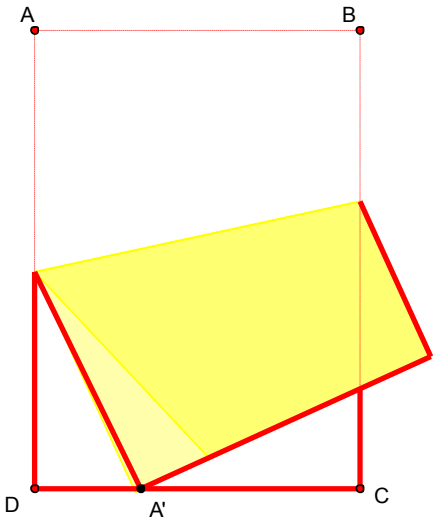
# Task 6: Comparing Triangles

Your job is to find 3 different-sized rectangles that fit Morgana's conditions. Attach the 3 rectangles to this sheet along with a brief note (1 or 2 paragraphs) to her describing:

- how you know each paper size produces triangles that are isosceles and congruent to each other
- how you found the 3 paper sizes
- two important things you discovered as you conducted your research

3. *Explore some more.* Start with another non-square rectangular piece of paper.

a) Instead of folding one corner onto the opposite corner (A onto C or B onto D), like you did in number 1, fold the paper so that A' lies somewhere on side $\overline{CD}$. There are three triangles showing from your fold. They are all right triangles. What else do you notice about these triangles?




b) Slide point A' along side $\overline{CD}$ and fold the paper so that A' is at a new location. Examine the triangles that are created. Slide point A' along side $\overline{CD}$ and fold the paper once again. Examine the triangles that are created. Use the chart below to record what you noticed and what you wondered about.

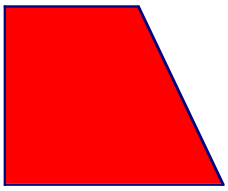
| | |
|---|---|
| **I noticed…** | |
| **I wondered…** | |

c) If you used a different sized piece of paper, do you think you would notice the same things you noticed in part b? Why or why not?

# Warm-up for Comparing Triangles

1. There are different ways to make quadrilaterals that are congruent to Quadrilateral A.

A

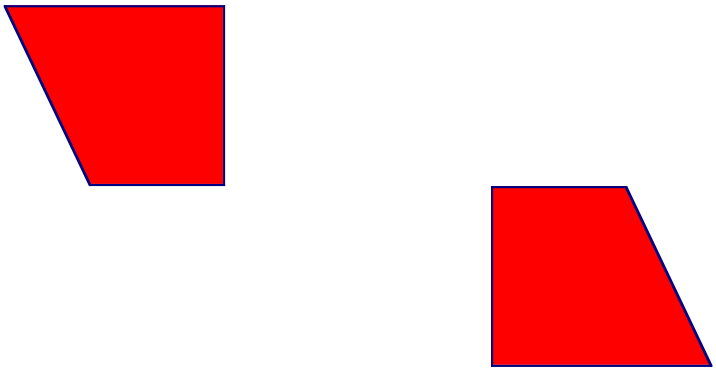


We can reflect Quadrilateral A to get the congruent quadrilateral on the left:

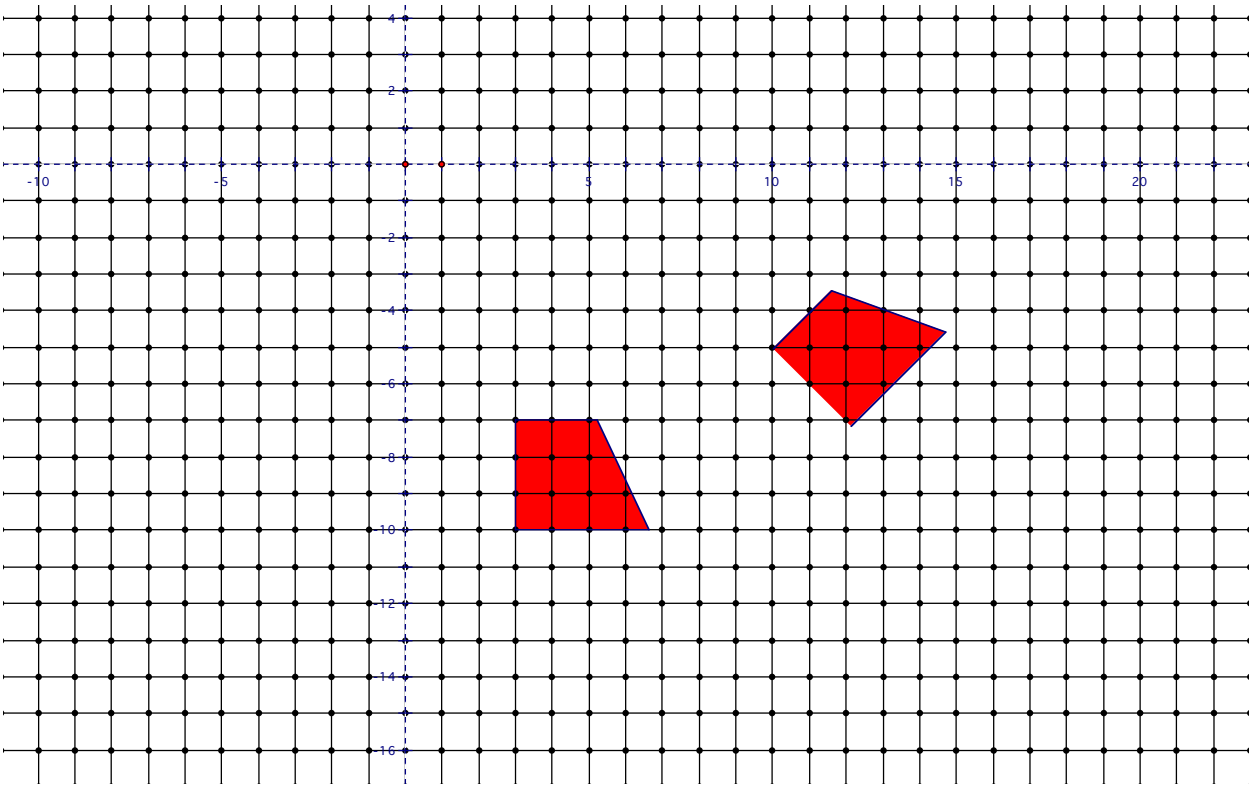
We can translate Quadrilateral A to get the congruent quadrilateral on the left:

We can rotate Quadrilateral A to get the congruent quadrilateral on the left:

# Warm-up for Comparing Triangles

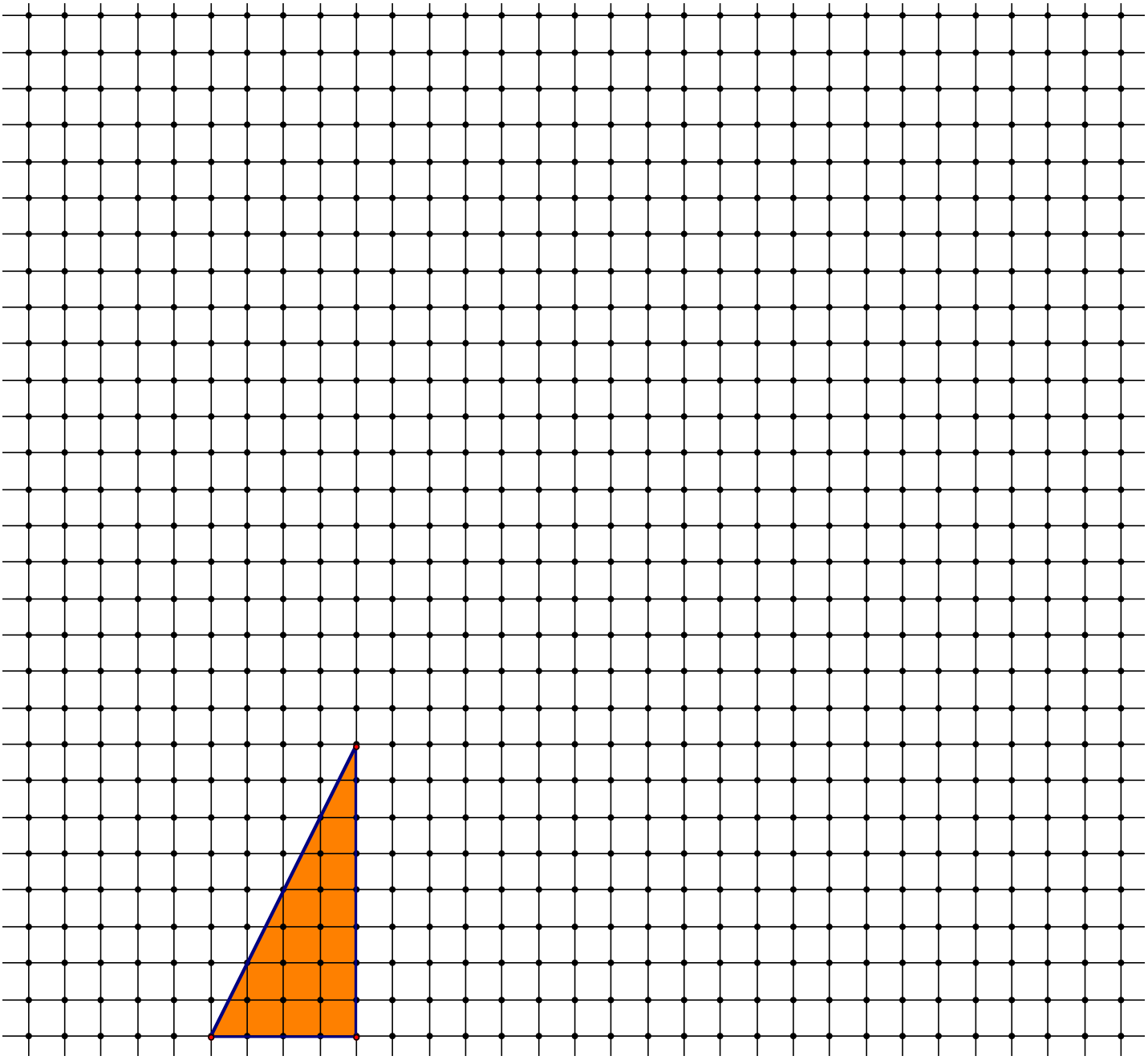


We also can make figures that are congruent to Quadrilateral A by using different combinations of reflections, translations, and rotations. Examine the grid below, and describe as precisely as you can how the congruent quadrilateral on the right was obtained from the quadrilateral on the left, with a combination of reflections, translations, and rotations:



# Warm-up for Comparing Triangles

2. A right triangle is drawn on the grid below. Draw a congruent triangle that is a translation of the right triangle. Draw a congruent triangle that is a reflection of the right triangle. Draw a congruent triangle that is a rotation of the right triangle



# Sentence Starters & Frames for Comparing Triangles

## Potential Sentence Starters and Frames

*Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.*

### Starters

When I fold the top left corner of a non-square rectangle to meet the bottom right corner, the triangles that are reflected over the fold are:

_____

For all sizes of non-square rectangles, if I fold the top left corner to meet the bottom right corner:

_____

Triangles EDC and FGC are alike in that they both_____

### Frames

When I fold the top left corner of a non-square _____ to meet the bottom right corner, the

_____ that are congruent.

Triangle EDC and Triangle FGC are _____

because _____

## Academic Language

*Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:*

- congruence; congruent
- reflection/reflect; translation/translate; rotation/rotate; reflected onto
- turn; slide; flip
- square; triangle
- non-square rectangle
- isosceles; isosceles triangles; isosceles right triangles
- diagonal
- line of reflection; line of symmetry
- similar; alike
- perpendicular bisector
- right angle
- line segment
- angle
- fold

# Word Chart for Comparing Triangles

Spanish, French, Portuguese
* = Cognate

| Words and Phrases | Academic Language Meaning | Everyday Language Version | Other Forms of the Word or Phrase | Related Words or Phrases | Translation | Examples of word use with students |
|---|---|---|---|---|---|---|
| **Congruent** | Two geometric figures in the plane are congruent if one can be obtained from the other by some combination of rotations, reflections, and translations | The same or equal in some way; same size and shape. | Congruently<br>Congruence | Congruous<br>Same shape | *Congruente<br>*Congruent(e)<br>Coincidente | |
| **Reflection** | A transformation in which each point is mapped to a corresponding point, which is an equal point from, and at right angles to, a mirror line; a map that transforms an object into its mirror image. | Mirror image. | Reflect<br>Reflects<br>Reflecting<br>Reflected | Copy<br>Duplicate<br>Symmetry | *Reflexión<br>*Réflexion<br>*Reflexão | |

**Mathematical Thinking:**
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

# Word Chart for Comparing Triangles

| Words and Phrases | Academic Language Meaning | Everyday Language Version | Other Forms of the Word or Phrase | Related Words or Phrases | Translation | Examples of word use with students |
|---|---|---|---|---|---|---|
| **Translation** | A transformation in which an object is moved to a new position without being turned or reflected. The translated image is the same size and shape as the object. | A change or conversion from one version to another; to modify. | Translate<br>Translates<br>Translating<br>Translated | Reposition<br>Relocation | *Traslación<br>*Translation<br>*Translação | |
| **Rotation** | A transformation in which a figure is turned so that each point on the image remains the same distance from a fixed point (2-D or 3-D) or line (3-D). | Turning around; moving clockwise or counter-clockwise. | Rotations<br>Rotating<br>Rotated | Turn<br>Moving in a circle | *Rotación<br>*Rotation<br>*Rotação | |
| **Isosceles Triangle** | A triangle that has two equal sides. The angles opposite these sides are also equal. | A triangle with two equal sides. | --- | --- | *Triángulo isosceles<br>*Triangle isocèle<br>*Triångulo isósceles | |
| **Rectangle** | A parallelogram with four right angles | ----- | Rectangular<br>Rectangles | Parallelogram<br>Quadrilateral<br>Quadrangle | *Rectángulo<br>*Rectangle<br>*Retângulo | |

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

# Word Chart for Comparing Triangles

| Words and Phrases | Academic Language Meaning | Everyday Language Version | Other Forms of the Word or Phrase | Related Words or Phrases | Translation | Examples of word use with students |
|---|---|---|---|---|---|---|
| **Turn** | --- | Movement around an axis or a center. | Turns<br>Turned<br>Turning | Rotation<br>Moving in a circle | Virar/Girar<br>*Tourner<br>Virar/Girar | |
| **Slide** | --- | Linear movement along a surface. | Slides<br>Slid<br>Sliding | Drift<br>Move along<br>Translate | Deslizar<br>Glisser<br>Deslizar | |
| **Flip** | --- | To turn over; to turn upright then down. | Flips<br>Flipped<br>Flipping | Reflection | Voltear<br>Retourner<br>Virar (cabeça para baixo) | |
| **Discover** | --- | To make known; obtain knowledge of. | Discovers<br>Discovered<br>Discovering<br>Discovery | Find<br>Determine | *Descubrir<br>*Découvrir<br>*Descobrir | |
| **Alike** | --- | In the same or similar manner, for, or degree. | Alikeness | Similar | *Similar<br>*Similaire/Semblable<br>*Similar | |

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

# Word Chart for Comparing Triangles

| Words and Phrases | Academic Language Meaning | Everyday Language Version | Other Forms of the Word or Phrase | Related Words or Phrases | Translation | Examples of word use with students |
|---|---|---|---|---|---|---|
| To Fold | --- | To lay one part over another part. | Folded<br>Folds | Enclose<br>Envelop<br>Double Over | Doblar<br>Plier<br>Dobrar | |

# Support Notes for Comparing Triangles

## "Turn"

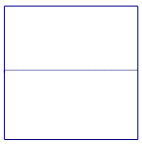
Be aware that the word "turn" has multiple meanings in English, and that it is used in two separate manners in the problem – once referring to rotations of geometric figures and another time to describe someone's "go" or turn in an order (*"Now it's your 'turn' to compare some shapes"*(top of page 2)).
Spanish and Portuguese have a cognate for "turn" – turno. However, "turno" in Spanish and Portuguese is not used to describe movement around a point (rotation).

## "Lies"

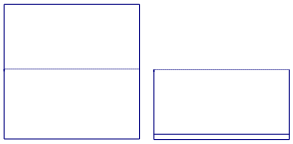
The word "lies" is used in the problem to refer to a location/position on a line segment (point A lies somewhere on side CD, page 5). It may be helpful to clarify the difference between "lie" -- as in "to tell a lie -- and "lie" – to "lie somewhere." A student who is familiar with the first meaning of the word but not the second might be led to believe the point referenced in the problem isn't actually on the line segment. That is, it's fake (a fraud).

# Comparando Triángulos

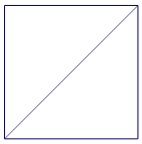
**Congruencia: Más de lo que encuentra el ojo.** A veces pensamos que dos figuras son congruentes porque parecen copias exactas, pero no siempre podemos confiar en nuestros ojos. En geometría, es importante ser preciso; decimos que dos figuras son congruentes si podemos obtener una con conbinaciónes de reflexiónes, translaciónes, y rotaciónes de la otra.
Lorna piensa que ella ha creado dos rectángulos congruentes dentro de un cuadrado.



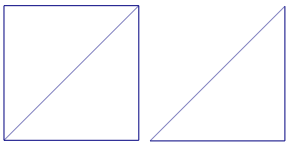
Sin embargo, conque ella sabía que figuras pueden parecer copias exactas cuando no lo son, ella decidió investigar más. Ella dobló su papel para ver si el rectángulo de arriba se podia reflexionar sobre el de abajo. Si los rectángulos son congruentes, uno de ellos cubrirá el otro exactamente. Cuando ella dobló el papel, eso no pasó! Los rectángulos parecían congruentes pero no lo éran!



Lorna decidió crear dos triángulos congruentes dentro del cuadrado. Dibujó una de las diagonales del cuadrado y exclamó, “Tengo dos triángulos isósceles que son congruentes!”

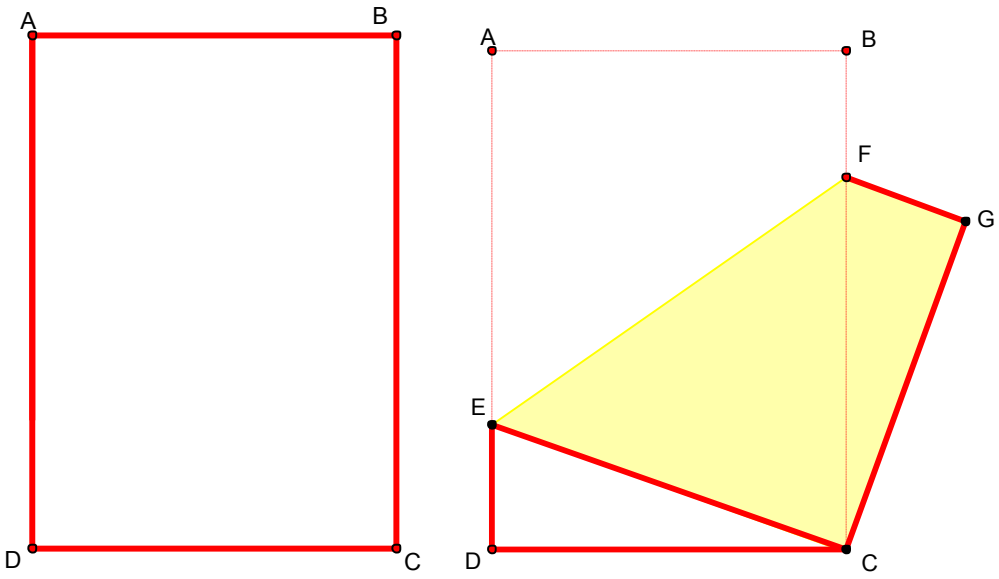


Ella recordó que tenía de asegurarse de que las figures sean realmente congruentes. Dobló el cuadrado en la diagonal y se dió cuenta que los triángulos cabían exactamente uno sobre el otro—éran congruentes porque uno era la reflexión del otro!



# Task 6: Comparando Triángulos

1. *Ahora te toca a tí comparar figuras.* Empieza con un papel rectángulo que no sea cuadrado, y asegúrate que és diferente que el rectángulo de tu vecino. Dobla el papel poniendo el punto A sobre el punto C como se vé en este diagrama.




a) Como se vé aqui, tu también verás algunos triángulos con tu papel. Con un marcador, traza los triángulos. ¿Cuantos encontrastes?

b) Compara los triángulos ΔEDC y ΔFGC en tu papel. ¿Como se parecen?

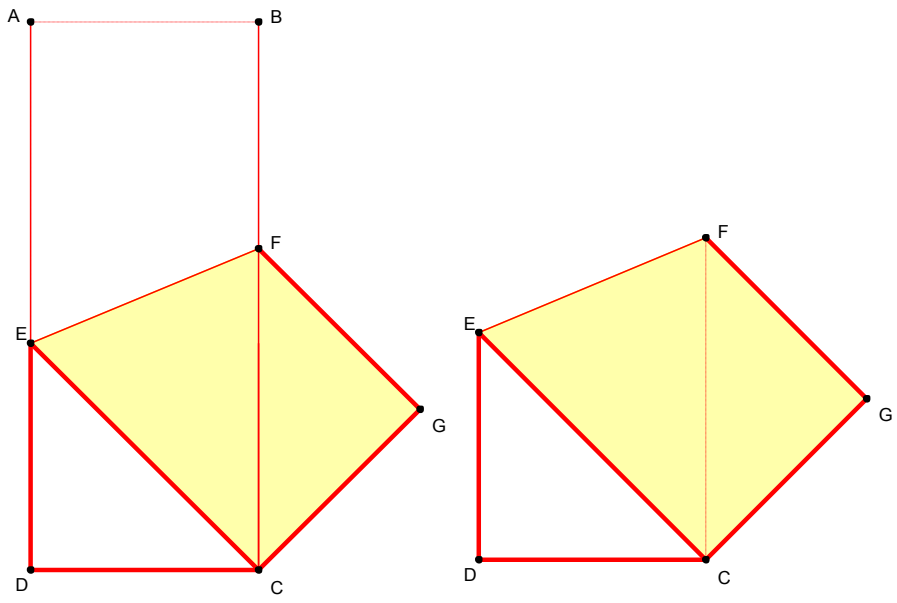
c) Tu vecino dobló un rectángulo diferente. Pregúntale lo que encontró. ¿Qué discubrió tu vecino sobre los triángulos ΔEDC y ΔFGC?

d) Julio piensa que los dos triángulos son congruentes, pero no sabe cómo probarlo. Ayuda a Julio demonstrar congruencia. Recuerdate de lo que pensó Lorna sobre congruencia-- dos figuras son congruentes si podemos obtener una con conbinaciónes de reflexiónes, translaciónes, y rotaciónes de la otra. Ayuda a Julio obtener el triángulo ΔEDC a base de reflexiónes, translaciónes, o rotaciónes del triángulo ΔFGC.

e) Mira el triángulo ΔCEF en nuestro diagrama. ¿Qué crées que és especial de este triángulo? Compara este triángulo en tu papel con el de tu vecino. ¿Qué notas?

2. *Investiga y ayuda a Morgana*. Morgana está empezando un pequeño negocio, haciendo y vendiendo materiales de geometría. Ella quiere tu consejo sobre un producto nuevo, papel isósceles para doblar.

Ella te dice: "Quiero vender papel que produce dos triángulos isósceles y congruentes. O sea, cuando uno dobla el punto A sobre el punto C, los triángulos que resultan, ΔEDC y ΔFGC, son isósceles y congruentes. No todos los papeles de forma rectángular producen esta clase de triángulos. Ayúdame a encontrar tres rectángulos de tamaños diferentes que créan una pareja de triángulos isósceles."



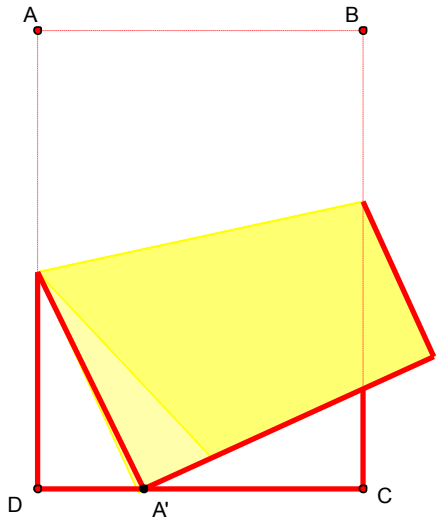
# Task 6: Comparando Triángulos

Tu tarea es encontrar tres rectángulos de tamaños diferentes para Morgana. Pega los tres rectángulos con este papel y escribe una nota (de uno o dos párrafos) explicando:

- cómo sabes que cada papel crea dos triángulos que son isósceles y congruentes
- cómo encontrastes los tres papeles rectángulares
- dos cosas importantes que descubristes hiciendo esta investigación

3. *Explora un poco más*. Empieza con otro papel rectángular que no sea un cuadrado.

   a) Esta vez, dobla el papel poniendo el punto A sobre algún lugar en el lado $\overline{CD}$. Doblando así se crean tres triángulos rectos. ¿Qué más notas sobre estos triángulos?




   b) Desliza el punto A por el lado $\overline{CD}$ y dobla el papel poniendo el punto A en otra locación. Investiga los triángulos que se crean. Desliza el punto A por el lado $\overline{CD}$ y dobla el papel otra vez. Investiga los triángulos que se crean. Usa el gráfico abajo para anotar tus observaciónes.

| | |
|---|---|
| **Me fijé que…** | |
| **Me pregunto si…** | |

   c) ¿Si usas papeles de tamaños diferentes, crées que notarías lo mismo? Explica tu respuesta.