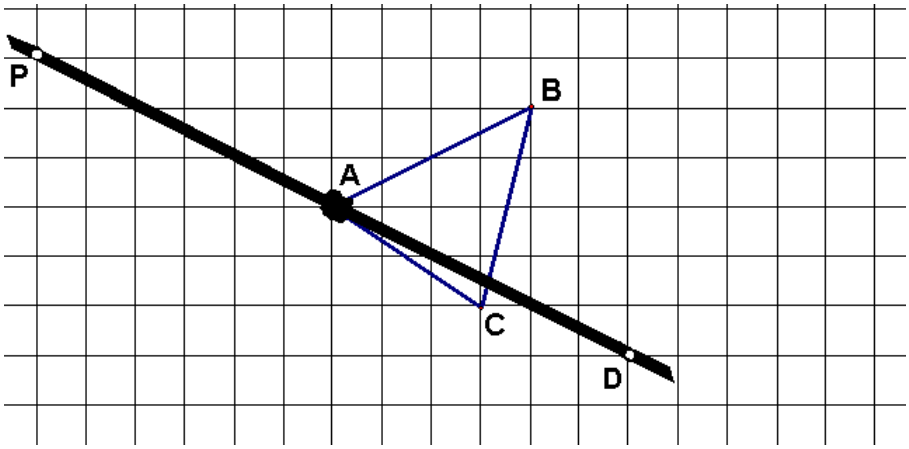
Dilating Figures

A stretcher is a rubber band that has been cut and then tied, creating a knot in the middle. A stretcher also has two dots, one on each side of its knot and equidistant from the knot—in this case, the dots are 2 inches from the knot.

In the questions that follow you will use a stretcher to create a new version of a triangle. The new version of the triangle is called a ***dilation*** of the original triangle. Stretchers are tools that can help you create ***dilations*** of any figure.

1. Follow the directions below to create a dilated version of ΔABC on Grid 1.
    a. Hold your stretcher so one of the dots is on point P.
    b. Pull the other end of the stretcher so the knot is on point A of the original triangle. (Check to make sure the dot is still directly on point P.) Now, make a mark on the grid where the second dot of the stretcher is located. Label this new point D.

Task 5: Dilating Figures



c. Repeat the process you just completed but this time move the stretcher so the knot is on point B of the original triangle. Use the second dot of the stretcher to help you locate point E of the new triangle.
d. Repeat this process one more time, placing the knot on point C to locate the position of point F of the new triangle.
e. Use a straightedge to connect the three vertices (D, E, and F) of your new triangle.

2. Compare the original triangle (ΔABC) to the dilated version of the triangle (ΔDEF).
   a. How are ΔDEF and ΔABC alike? (list 2 or 3 mathematical ways in which they are alike)

   b. How are ΔDEF and ΔABC different? (list 2 or 3 mathematical differences)

   c. On Grid 1, draw line segments to represent the distances from the knot to each dot for the three positions of the stretcher. For example, draw a segment that connects points P and B, and then another that connects points B and E. Do this for all 3 vertices of the triangle.

   d. Compare segment PA's length to that of segment AD. Do that same for segment PB and segment BE. Now do the same for segment PC and segment CF. What do you notice about the relationship between the lengths?

Task 5: Dilating Figures

3. On Grid 1, draw a new point four grid spaces below the original point P and label it P_2. Without using the stretcher, predict what a dilated version of ΔABC will look like and where it will land on the grid when you use P_2 instead of P.
    a. Use a different color of pencil or marker and draw your predicted triangle on the grid (but don't use the stretcher!).
    b. How did you decide where to put the new dilated triangle and what it should look like? Describe what you thought about as you made your prediction.

    c. Check your prediction. Use P_2 and your stretcher to create a new dilated triangle based on ΔABC. Use a different color of pencil or marker to draw this triangle. Was your prediction correct? Explain.

Task 5: Dilating Figures

4. Take a look at the Geometer's Sketchpad file "dilations.gsp" available in the Fostering Geometric Thinking CD.[1] In this file, ΔDEF is the dilated version of ΔABC using point P as the center of dilation. Move the point P around on the grid and explore what happens to the dilated triangle (ΔDEF).
    a. What do you notice?

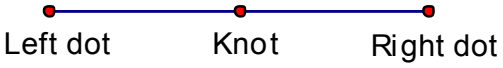
    b. What questions come to mind?

5. Return to Grid 1 and draw a third point P three grid spaces to the right of P. Label this new point P_3. Without using the applet or your stretcher, construct a new dilated triangle based on P_3 and ΔABC. Use a different color of pencil or marker to draw this triangle. How did you decide where the new triangle should be on the grid?

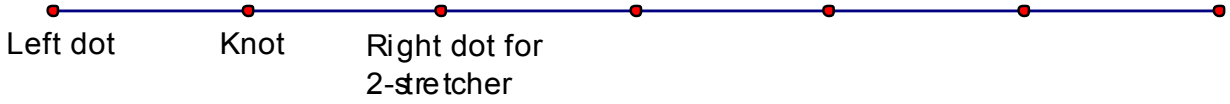
6. Maria made the dilated triangle shown on Grid 2 using a stretcher like yours. Someone erased her original triangle. Without using a stretcher, reconstruct her original triangle on Grid 2. Describe how you figured out what her original triangle would look like and where it would be located.

[1] If you do not have access to Geometer's Sketchpad software, you can find a web-based applet appropriate for this problem at http://www.geometric-thinking.org/dilations.htm.

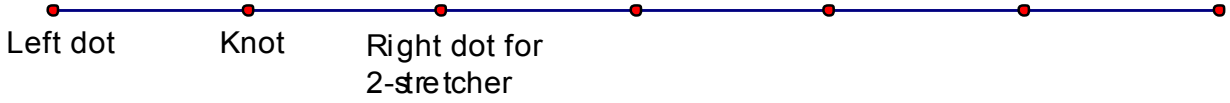
7. Here is a drawing of the kind of stretcher you've been using.



You probably figured out that this stretcher doubles the lengths of the sides of triangles like ΔABC. So, we could call it a 2-stretcher. Suppose you wanted to make a 3-stretcher – that is, a stretcher that triples the lengths of the sides of ΔABC. Where would you put the Right dot for a 3-stretcher? Explain.

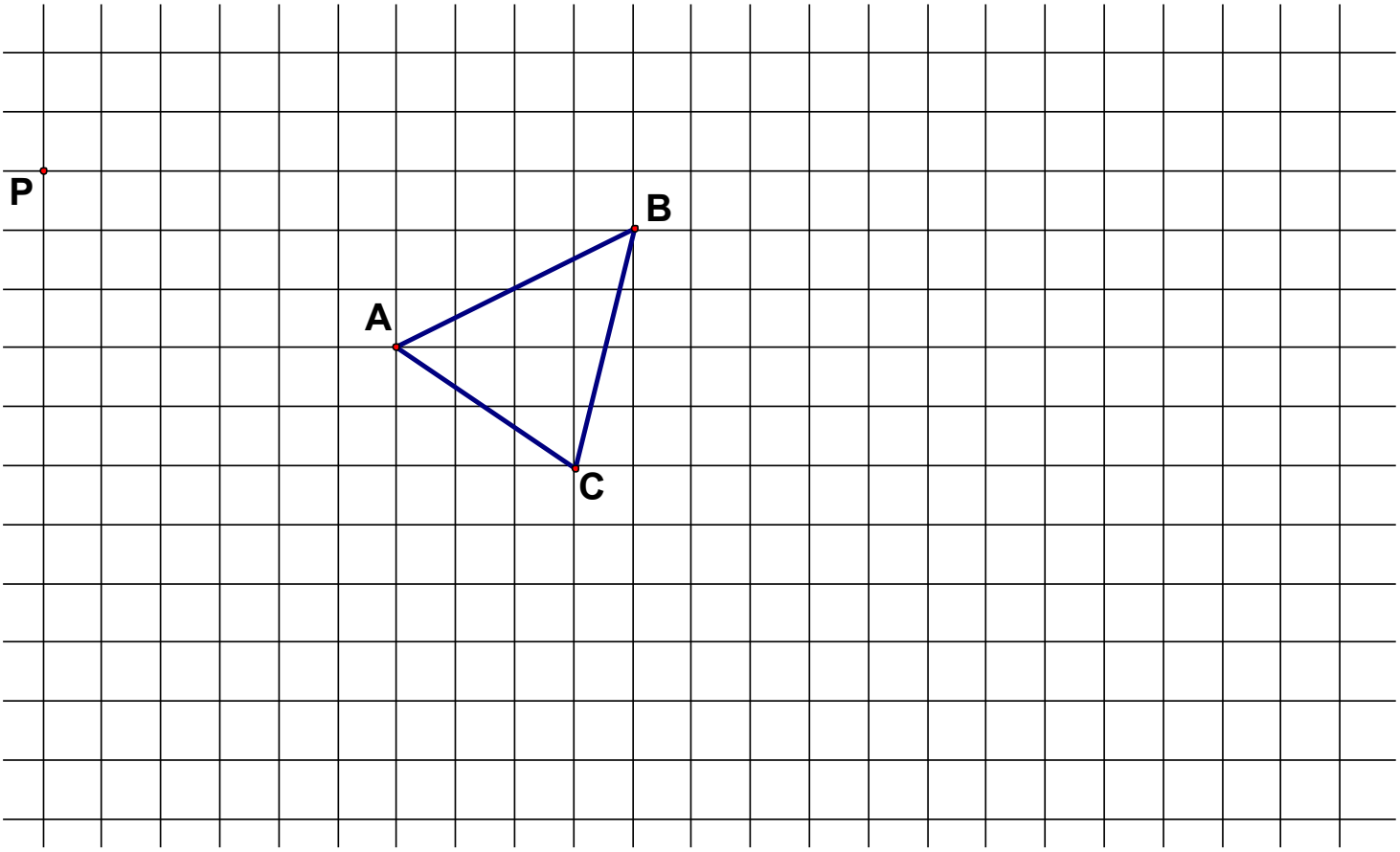


Where would you put the Right dot for a $\frac{1}{2}$-stretcher? Explain.



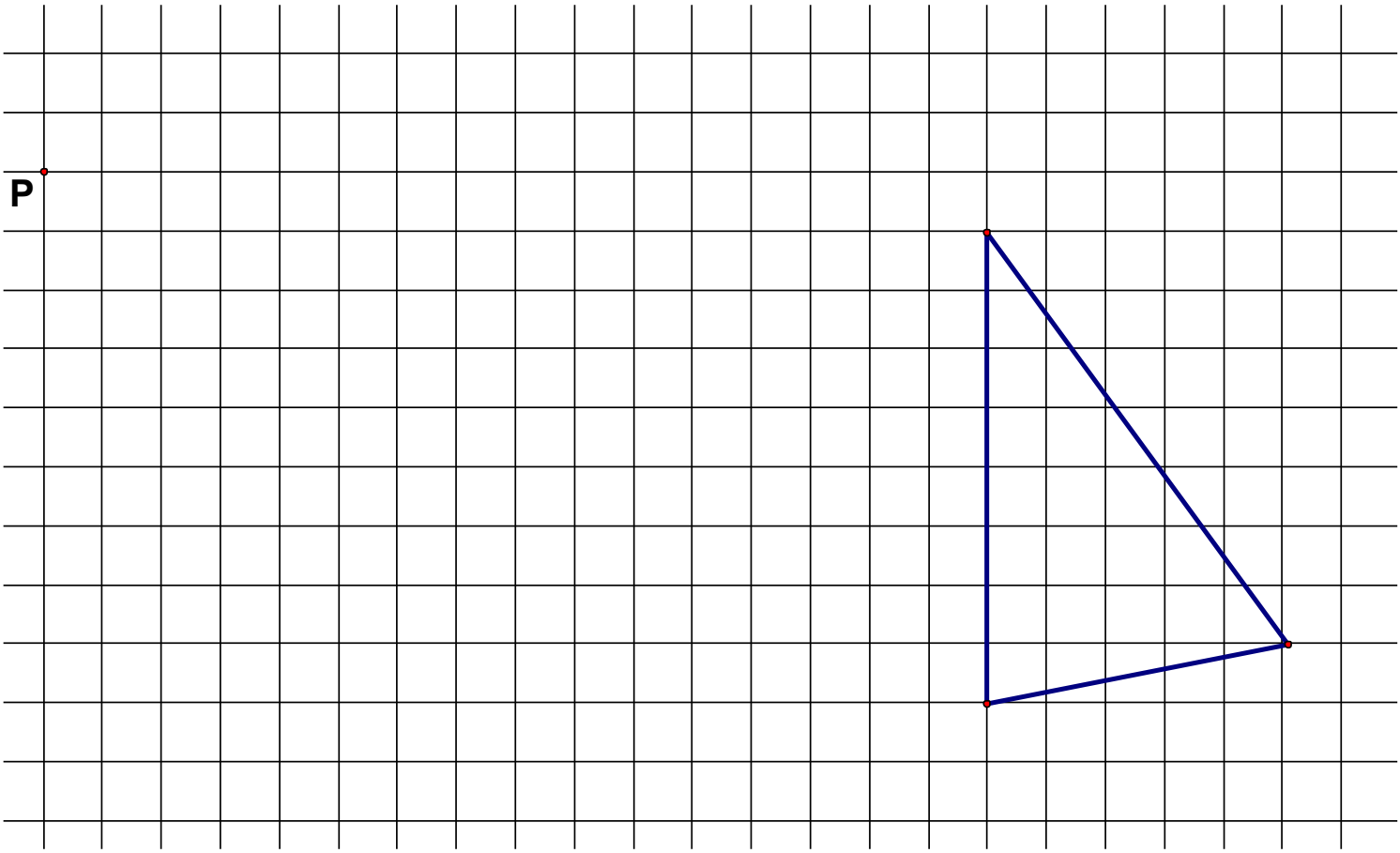
Task 5:
Dilating Figures

Grid 1



Task 5:
Dilating Figures

Grid 2



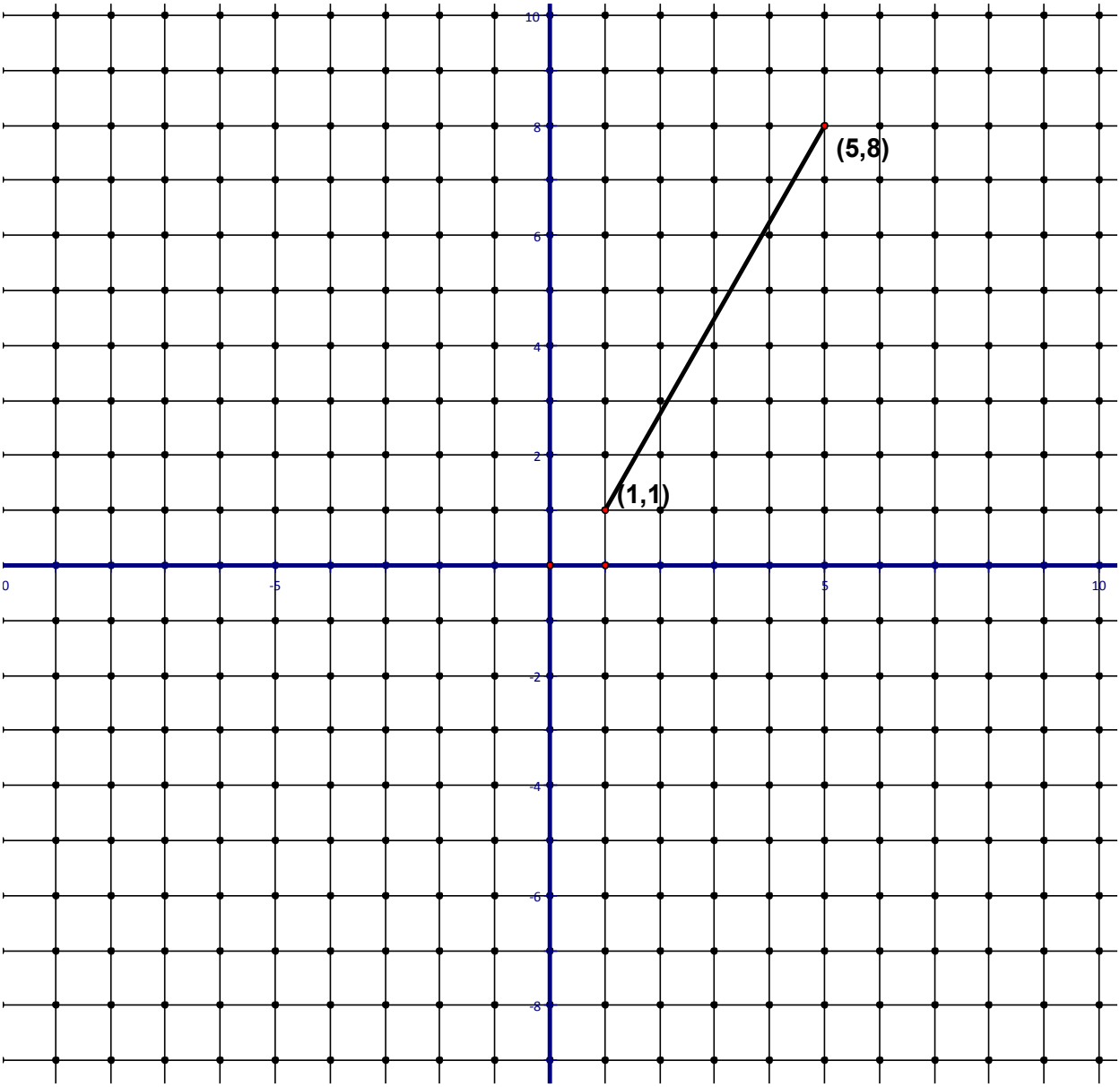
Instructions for Creating Stretchers

Each student or pair of students will need a stretcher. To create a stretcher, use rubber bands 2.5 inches in diameter or greater and complete the following steps:

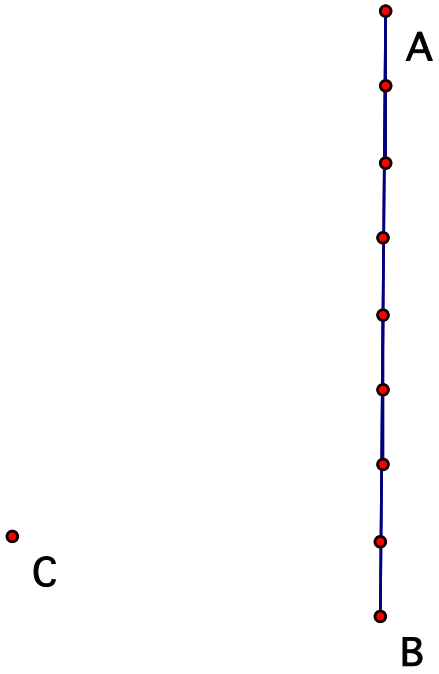
a. Cut each rubber band.
b. Tie a knot close to the middle of the rubber band. (The knot will be used to trace the original figure.)
c. Use a permanent marker to place a dot 2 inches to the left of the knot, and another dot 2 inches to the right of the knot. (One of these dots will be lined up with the anchor point, point P, and the other dot will indicate where the vertices of the dilated figure are located.

Warm-up for Dilating Figures

1. A line segment connects the points (1,1) and (5,8) on the grid below. What are the coordinates of the midpoint of this line segment? Explain how you found the midpoint and its coordinates.

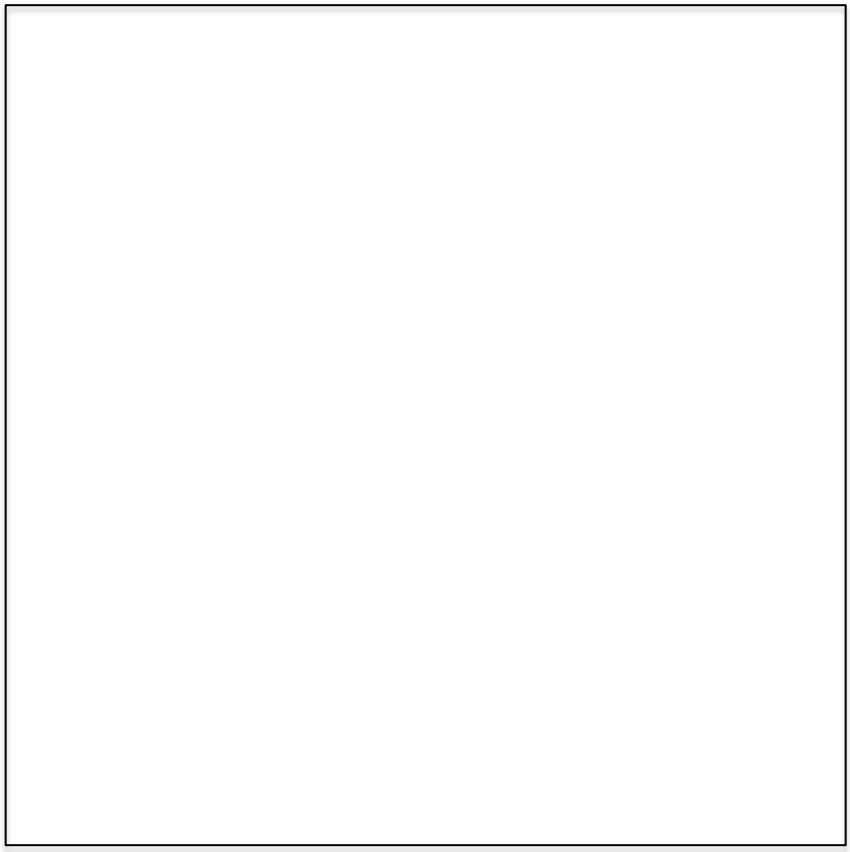


2. Below are a point C and a line segment $\overline{AB}$. The line segment is 8 cm long and has been divided into 8 equal parts.



Take a ruler and place one end on C, and measure the distance to one of the 8 marked points on $\overline{AB}$. Then double this distance and mark a new point at this new location. We have done it for point B, in the picture below. We measured and found the distance from C to B to be very close to 5 cm, so we doubled that to find point B', a distance of 10 cm from C.

Warm-up for Dilating Figures



Imagine that you have repeated the same process for the other 7 points on $\overline{AB}$. Predict what the new 8 points will look like. Will they also be on a straight line? How far apart will they be? Now repeat the process we used to locate B', for each of the 7 other points shown on $\overline{AB}$. Connect the 8 resulting points. Do you get a new line segment? How long is it?

Sentence Starters & Frames for Dilating Figures

Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.

Starters

When I dilate a triangle, I notice that:
The angles of the original triangle and of the dilated image of that triangle _____

_____

The lengths of the sides of the original triangle and of the dilated image of that triangle _____

The distance from the center of dilation (the point P) to the vertices of the original triangle and from the

center of dilation (the point P) to the dilated image of that triangle_____

_____

(Part 3 prediction)
I predict that if I move the center of dilation (the point P) four grid spaces down, then the dilated image of

triangle ABC will _____

_____

Frames

The _____ of the original triangle and of the dilated image of that triangle remain the same.

The _____ of the _____ of the original triangle and of the dilated image of that

triangle are proportional.

Sentence Starters & Frames for Dilating Figures

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- Stretcher, knot
- dilation of the original triangle; dilated triangle; dilated version of the triangle; dilate
- center of dilation
- the side lengths double
- the distances double
- scale factor
- similar triangles; similarity; similar figures
- predict, reconstruct, locate
- orientation of the triangle
- angles
- side length; line segment
- spaces

Word Chart for Dilating Figures

Spanish, French, Portuguese
* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Dilation	A transformation that enlarges or reduces a geometric figure proportionally	Enlargement or expansion (e.g., of the pupils)	Dilate Dilates Dilating Dilated	Amplify Enlarge Make bigger	*Dilatación *Dilatation *Dilatação	
Vertices	The points where the sides of a geometric figure intersect.	--	Vertex	Intersection Corner	*Vértices Sommets *Vértices	
Similar	For polygons: having corresponding angles congruent and corresponding sides in proportion	Alike: having the same characteristics	Similarity Similarly Similarities	Alike Matching Proportional	*Similar *Similaire *Similar	

Word Chart for Dilating Figures

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Equidistant	Being the same distance from two other points.	Equally distant.	Equidistantly	In between	*Equidistante *Equidistant(e) *Equidistante	
Line Segment	A straight line that links two points without extending beyond them.	Unbroken and bounded portion of line.	Segmented line Line segments Segmenting lines	Part of line Portion of line Section of line	*Segmento de línea *Segment *Segmento de linha	
Predict	To make a precise guess based upon reason and study of a situation.	To suggest what may happen (e.g. weather); to tell something in advance.	Prediction Predicted Predicting Predicts	Forecast Foresee Anticipate	*Predecir/ Prever *Prédire/ Prévoir *Predizer/ Prever	
Reconstruct	To partition a geometric shape and reorganize as to alter its appearance.	To reassemble/reorganize the parts or pieces of something.	Reconstructed Reconstructing Reconstructs	Rebuild Reassemble Reorganize	*Reconstruir *Reconstruire *Reconstruir	

Word Chart for Dilating Figures

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Knot	---	A piece of material folded and tied upon itself.	Knots Knotted	--	Nudo Nœud Nó	
Locate	---	To identify or discover the place of.	Locates Locating Location Located	Find Discover Identify	Ubicar/*Localizar Trouver/*Localiser *Localizar	
Spaces	---	A blank portion or area; a linear unit on a grid.	Space Spaced	Room Empty area	*Espacios Places/*Espaces *Espaços	

Support Notes for Dilating Figures

I. Espaces vs. Places in French

While the French cognate "espaces" can be used as a translation for "spaces," the word "places" is more frequently used to describe empty area or expanses of empty area.

II. To locate (Spanish & French)

While cognates can be used to express the English verb, "to locate," in Spanish and French (i.e., localizar, localiser), the words "ubicar," as well as "encontrar," (Spanish) and "trouver" (French) are more frequently used to express the same idea.

III. Sommets (French)

In French, the best translation for vertices is "sommets," as in "summits." "Sommets" in French, as in English, is also used to refer to peaks of structures or features of landscape (e.g., mountain). It might be useful to differentiate between the uses (i.e., a vertex doesn't have to be on top) and also expound on how the uses are similar (i.e., the convening of two lines).

IV. "Line" vs. "Line Segment"

Students are asked in this task to create "line segments." It may help to clarify the difference and similarity between a "line" and a "line segment." That is, a line segment is an unbroken and bounded portion of a line; a line is infinite.

V. "Knot" vs. "Not"

Be aware that some students may not be familiar with the word "knot," but are likely familiar with "not" – which is pronounced the same way. Addressing the difference between the two terms might help to prevent students from confusing the two words.

Dilatando Figuras

Un "stretcher" es una banda elástica que se ha cortado y atado, creando un nudo en el medio. Un *stretcher* tambien tiene dos puntos, uno en cada lado del nudo y a la misma distancia del nudo—en este caso, los puntos están a dos pulgadas del nudo.

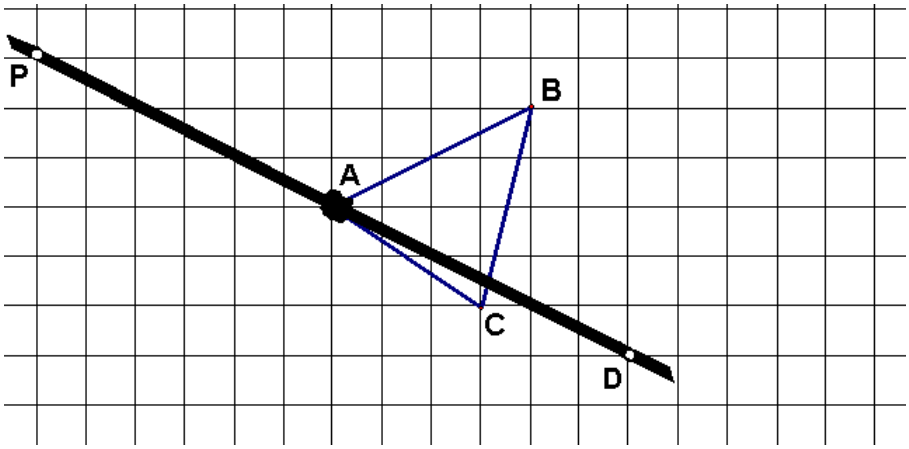
En los problemas siguientes, usarás un *stretcher* para crear una nueva versión de un triángulo dado. Esta nueva versión se llama una **dilatación** del triángulo original. Los stretchers son herramientas que pueden ayudar a crear dilataciones de cualquier figura.

1. Sigue estas direcciones para crear la dilatación del ΔABC en el papel cuadriculado #1 y así hallar los puntos D, E, F del nuevo triángulo. Llamaremos a este triángulo dilatado: ΔDEF.

    a. Coloca tu *stretcher* de tal forma que uno de los dos puntos quede encima del punto P.

    b. Estira la otra punta del *stretcher* colocando el nudo encima del punto A en el triángulo original. (Asegúrate que el primer punto todavía está directamente encima del punto P). Ahora, marca en el papel el sitio donde está el segundo punto del *stretcher*. Llama a este nuevo punto D.

Task 5:
Dilatando Figuras



c. Repite este proceso pero esta vez coloca el *stretcher* con el nudo sobre el punto B del triángulo original. Usa el segundo punto del *stretcher* para ayudarte a localizar el punto E del nuevo triángulo.

d. Repite este proceso una vez más, poniendo el nudo sobre el punto C para localizar la posición del punto F del nuevo triángulo.

e. Usa algún material recto para conectar los tres vértices (D, E, y F) de tu nuevo triángulo.

2. Compara el triángulo original (ΔABC) con su dilatación (ΔDEF).

a. ¿En qué se parecen los ΔDEF y ΔABC? (escribe dos o tres formas que describan en forma matemática en que se parecen los triángulos)

b. ¿De qué manera son los triángulos ΔDEF y ΔABC diferentes? (describe dos o tres diferencias matemáticas).

c. En el cuadriculado #1, dibuja segmentos de línea que representen las distancias desde el nudo hasta cada punto en las tres posiciones diferentes en que pusiste el *stretcher*. Por ejemplo, dibuja un segmento de línea del punto P al punto B, y otro del punto B al punto E. Haz esto con cada uno de los tres vértices del triángulo original y los del dilatado.

d. Compara la longitud del segmento PA con la longitud del segmento AD. Haz lo mismo para los segmentos PB y BE. Ahora, repite lo mismo para los segmentos PC y CF. ¿Qué notas acerca de la relación entre las longitudes?

3. En el cuadriculado #1, dibuja un punto nuevo cuatro espacios abajo del punto P y llámalo P2. Sin usar el *stretcher*, haz una predicción de cómo se verá la dilatación del ΔABC y dónde estará localizada en el papel cuadriculado cuando usas P2 en vez de P para crear la dilatación del triángulo original.

    a. Usa un lápiz o un marcador de color diferente y dibuja el triángulo donde crees que va a quedar en el papel cuadriculado (¡Pero no uses el stretcher!).

    b. ¿Cómo decidiste dónde dibujar el triángulo dilatado? ¿Cómo supiste la forma que tendría el triángulo dilatado? Describe lo que estabas pensando cuando estabas haciendo tu predicción.

    c. Verifica tu predicción. Usa el punto P2 y tu *stretcher* para crear la dilatación del triángulo ABC. Usa un lápiz o un marcador de color diferente para dibujar este triángulo. ¿Hiciste una predicción correcta? Explica como sabes que estaba correcta o no.

Task 5: Dilatando Figuras

4. Revisa el archivo "dilations.gsp" del Geometer's Sketchpad que se encuentra dentro del CD de "Fostering Geometric Thinking"1. En este archivo, ΔDEF es la dilatación de ΔABC usando el punto P como el centro de dilatación. Mueve el punto P a diferentes lugares en el cuadriculado y explora que pasa con el triángulo dilatado (ΔDEF).

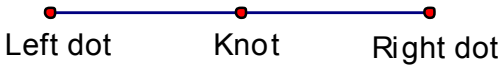
    a. ¿Qué notas?

    b. Qué preguntas te vienen a la mente (o se te ocurren)?

5. Vuelve al cuadriculado #1 y dibuja un tercer punto tres espacios en la derecha de P. Llama este punto P3. Sin usar el applet o tu *stretcher*, construye la dilatación de ΔABC cuando P3 es el centro de dilatación. Usa un lápiz o marcador de otro color diferente para dibujar este triángulo. ¿Cómo decidiste dónde poner el triángulo dilatado en el cuadriculado?

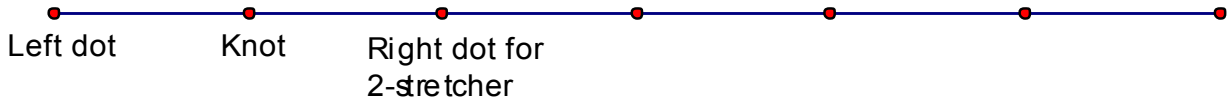
6. Maria dibujó el triángulo que se muestra en el papel cuadriculado #2 usando un *stretcher* como el tuyo. Alguién borró su triángulo original. Sin usar el *stretcher*, reconstruye su triángulo original en el papel cuadriculado #2. Describe como supiste la forma que debería haber tenido el triángulo original y en donde debía haber estado colocado el el cuadriculado #2.

7. Aquí hay un dibujo de el tipo de *stretcher* que has estado usando.



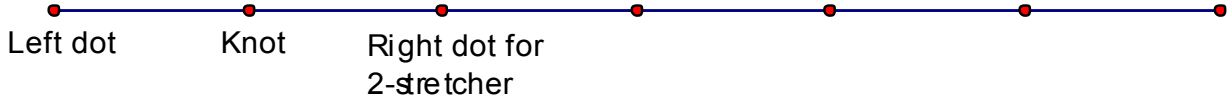
Punto izquierdo nudo punto derecho

Probablemente te has dado cuenta de que este *stretcher* dobla las longitudes de cada lado de triángulos como el ΔABC. O sea, lo podríamos llamar un *stretcher*-de-dos. Imagina que quieres construir un stretcher-de-tres—esto quiere decir un *stretcher* que triplique las longitudes de los lados del ΔABC. ¿Dónde pondrías el punto derecho por un stretcher-de-tres? Explica.



punto izquierdo nudo punto derecho para
un strecher-de-dos

¿Dónde pondrías el punto derecho de un *stretcher*-de-½? Explica.



Punto izquierdo nudo punto derecho para un *stretcher*-de-do