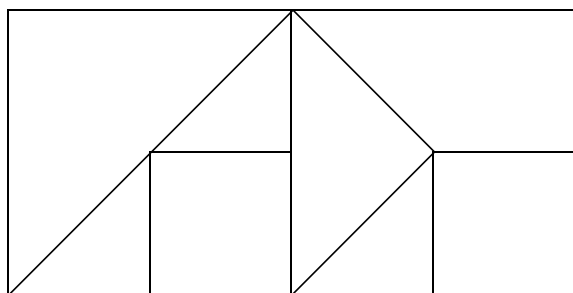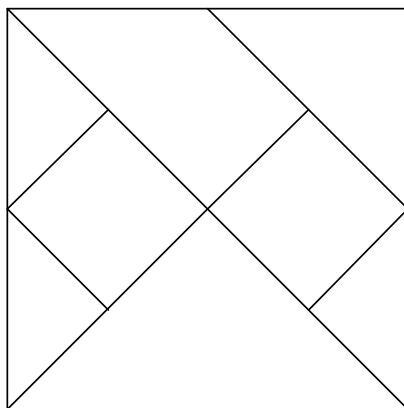## Puzzling with Polygons

Milo's teacher gave him a square and asked him to cut it into pieces to create a puzzle. After experimenting with different ways to cut his square into pieces, Milo said to Sarah: "Look, Sarah, I cut the square into 8 pieces and then rearranged those pieces to form a nonsquare rectangle!"

1. Sarah offers a challenge to Milo. "Okay," she says, "you were able to reassemble your 8---piece square puzzle to fit this rectangle exactly...
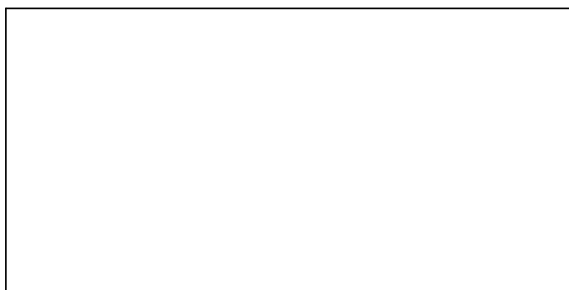
… but, what if I take that rectangle and scale it up so that it has 1.5 times the width and 1.5 times the height…
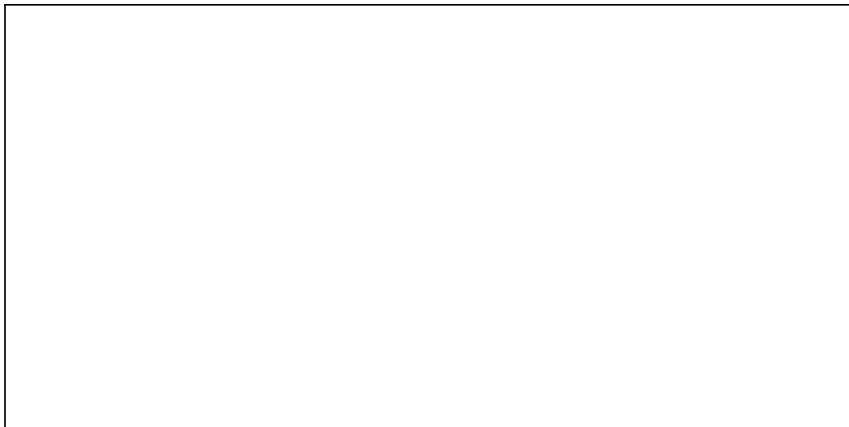
"Can you dissect this rectangle into 8 puzzle pieces and reassemble all of them to make a square?"

a) Help Milo meet Sarah's challenge by cutting this rectangle (there is a copy of the rectangle at the back of this packet) into 8 pieces and rearranging the pieces to form a square. Paste or draw the square you formed below:

b) Without measuring,

    i)   state the relationship between the side length of Milo's original square and the side length of the square you just created.

    ii)   state the relationship between the area of Milo's original square and the area of the square you just created.

1. Now Milo wants to pose a challenge to you! Your challenge is to cut the square at the back of this packet into only 6 pieces that can be rearranged to form the nonsquare rectangle below.
   a) Can you combine the shapes in Milo's 8---piece puzzle to form a 6---piece puzzle out of the square at the back of this packet that can be rearranged to form the nonsquare rectangle, below?
   b) Can you make a different 6---piece puzzle out of the square that can be rearranged to form the nonsquare rectangle shown below?
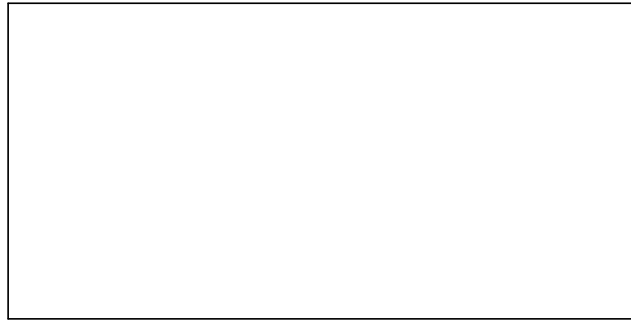
c) Choose one of the two six---piece puzzles you created in parts a and b and show how your square puzzle could be reassembled to make a nonsquare rectangle. Draw your puzzle pieces in the square and nonsquare rectangles below.
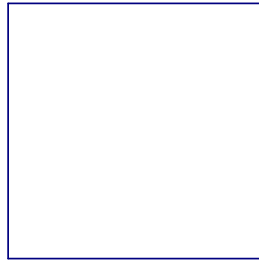
d) For the same puzzle, describe in words how your square could be reassembled to make a nonsquare rectangle

e) What do you know about each of your puzzle pieces for the puzzle you described in parts c and d? Complete the table.

| Picture of puzzle piece	Name the shape	List the properties of that shape

f) Sarah says to you, "I know all rectangles have 4 right angles, 4 straight sides, and opposite sides that are the same length. It may *look* like your final rectangle has those properties but I need you to convince me so that I know for sure." (You might want to use what you know about the properties of your puzzle piece shapes (part c) to help convince Sarah.)

   i. Convince Sarah your nonsquare rectangle has 4 right angles.

   ii. Convince Sarah your nonsquare rectangle has 4 straight sides.

iii.   Convince Sarah your nonsquare rectangle has opposite sides that are the same length.

Extension:

Use another square from the back of this packet and make an 8---piece square puzzle that, like Milo's, also can be rearranged to form a nonsquare rectangle. Your puzzle should not look exactly like Milo's though. Create your puzzle so that the pieces are symmetrical about one of the *square's* diagonals.
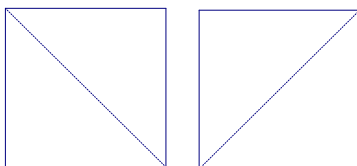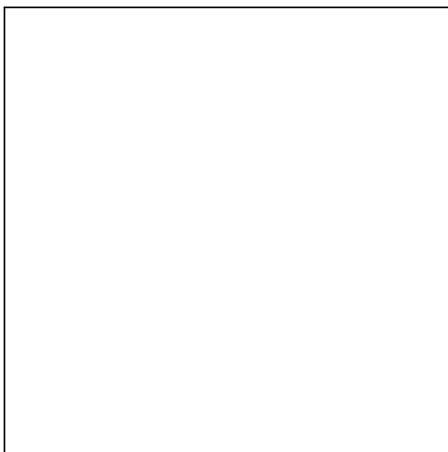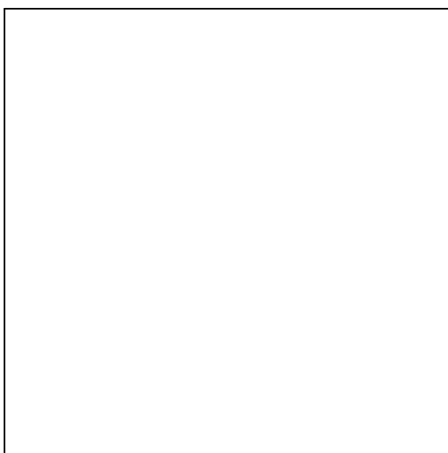
Here's an example of a square with 2 puzzle pieces symmetrical about its diagonal.
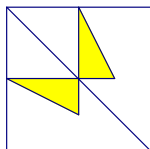
a)   Draw a picture of your square, showing how you divided it into puzzle pieces that were symmetrical about one of its diagonals. Then draw a picture of how you rearranged the puzzle pieces to form a non---square rectangle.

b)   Name all of the angles in your pieces without measuring.

c)   Convince Sarah that your final shape is a non---square rectangle.

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Shape Templates
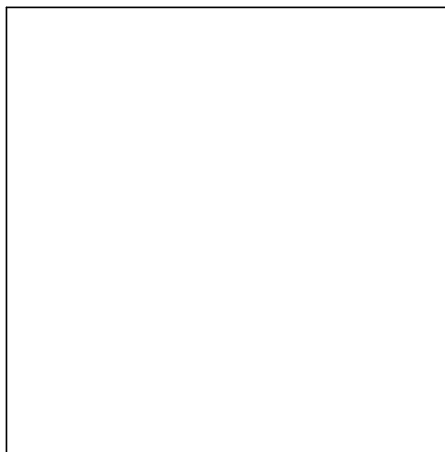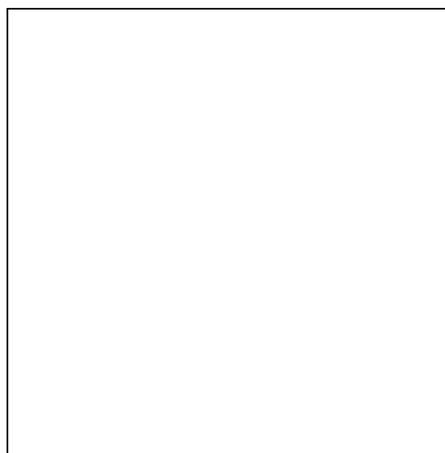
Warm---up for Puzzling with Polygons

1. Below is a right triangle. Dissect it into three pieces so that you have 1 rectangle and 2 triangles.
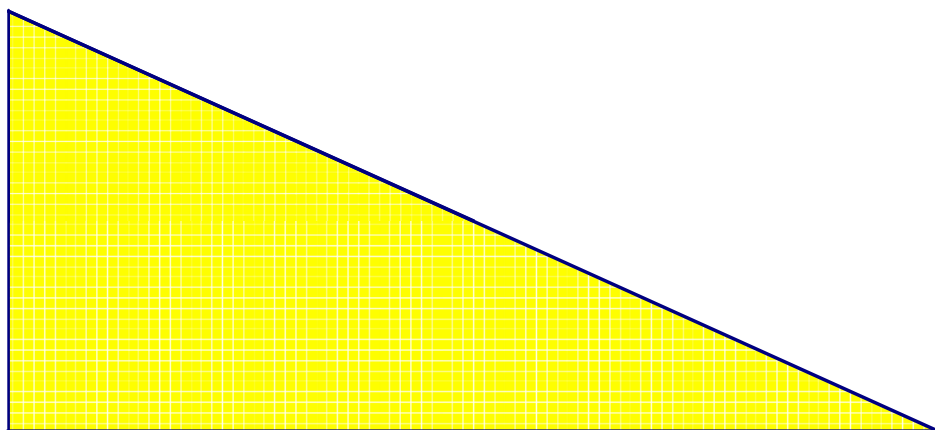
2. Now dissect this right triangle into three pieces so that you have 1 rectangle and 2 triangles—with one more condition: *the three pieces can be reassembled to make a rectangle*:

3. Dissect this square into 4 triangles, which can be reassembled to make a non---square rectangle:

Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you cre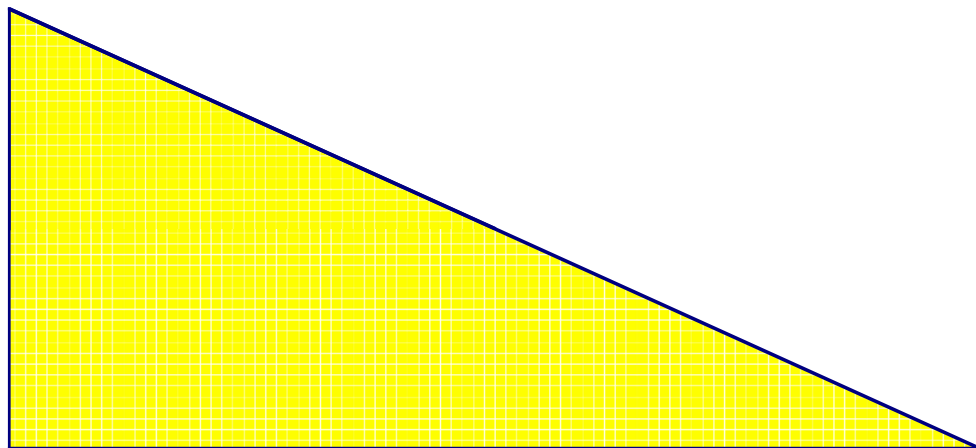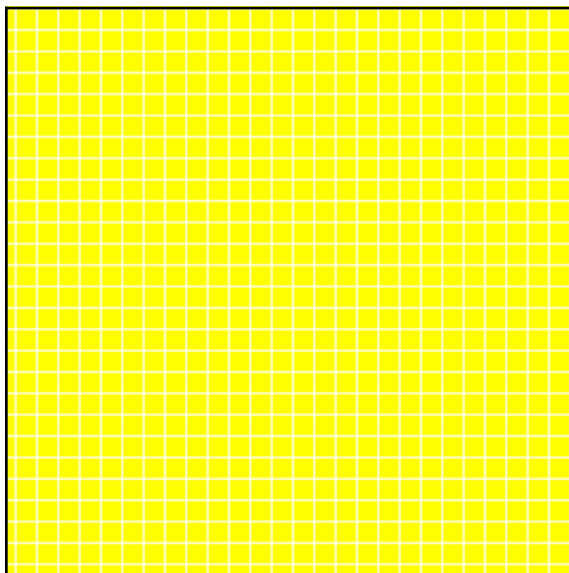ate) would best support students' learning. You may find that the starters and frames vary in level of difficulty, and plan to provide them to students accordingly.

Starters
(Question 1)
To make sure that I will be able to create a square, when I cut the rectangle into pieces I need to make sure

that _____

_____

(Question 2f)
I know that my non---square rectangle has 4 right angles because_____

I know that my non---square rectangle has 4 straight sides because _____

I know that opposite sides of my non---square rectangle are the same length because

_____

_____

Frames
(Question 1)
To make sure that I will be able to create a_____, when I cut the rectangle into pieces I need to

make sure that _____

(Question 2f)

I know that my non---square rectangle has 4_____angles because _____

_____

I know that opposite sides of my non---square rectangle are the same_____because _____

_____

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- square, non---square rectangle, rectangle
- dissect, reassemble, rearrange, cut
- scale up, width, height
- measuring
- side length, area, diagonal, symmetrical, proportional
- right angle

Mathematical Thinking: SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Puzzling with Polygons

Word Chart for Puzzling with Polygons

Spanish, French, Portuguese

* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Square	A 4---sided regular polygon with all sides equal and all internal angles 90 degrees; two---dimensional	A rectangle with all four sides equal	Squares Squarely	Equal sides Congruent sides	Cuadrado Carré Quadrado	
Rearrange	To partition a geometric shape and reorganize as to alter its appearance.	To put something into a new order.	Rearranges Rearranged Rearranging	Change Reconstruct Reorganize Reorder	Reorganizar *Réarranger *Rearranjar	
Reassemble	To partition a geometric shape and reorganize as to alter its appearance.	To reconstruct the parts or pieces of something.	Reassembled Reassembling Reassembles	Rebuild Reconstruct	*Reensamblar Reformer Reagrupar	

Mathematical Thinking: SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Puzzling with Polygons

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Non---square rectangle	A quadrilateral with four right angles, opposite sides of the same length, and adjacent sides of different lengths	------	Non---square rectangles	------	Rectángulo (que no séa cuadrado) Rectangle (qui ne soit pas carré) retângulo (que não seja quadrado)	
Scale up	To increase proportionately.	To expand in scope and size.	Scaling up Scaled up Scales up	To augment To enlarge To expand	Ampliar proporcionalmente Amplifier proportionellement Amplificar proporcionalmente	
Dissect	To partition a geometrical figure by straight lines.	To cut open and examine the inside of; to cut apart or separate.	Dissects Dissected Dissecting Dissection	Break up Partition Divide Slice	*Diseccionar *Disséquer *Dissecar	
Measuring	Assigning scalable units or value to a component of a geometrical shape based on its size (e.g., length, width)	Assigning numerical values based on an object's physical properties.	Measure Measures Measured Measurement	Assessing Calculating Evaluating Gauging	Medir *Mesurer Medir	

Mathematical Thinking: SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Puzzling with Polygons

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Area	The number of square units contained in the interior of a figure; the extent of a two‑‑‑dimensional surface enclosed within a boundary	The space something occupies, two‑‑‑dimensional	‑‑‑‑‑‑	Enclosed Space Two‑‑‑dimensional size	*Area *Aire/ Superficie *Área	
Symmetrical	A shape that becomes exactly like a corresponding shape after a reflection, translation or rotation.	Equal appearance about a dividing line.	Symmetry Symmetric Symmetries	‑‑‑‑‑‑	*Simétrico *Symétrique *Simétrico	
Cut	To separate a geometric figure into pieces.	To make an opening, incision.	Cuts Cutting	Chop Slice Separate Sever	Cortar Couper Cortar	
Proportional	Having a constant ratio to another quantity.	Corresponding in size or amount to something else.	Proportion Proportionally	Commensurate Equivalent Uniform	Proporcional Proportionelle Proporcional	

3

Mathematical Thinking: SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Puzzling with Polygons

| Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Template	------	Something that serves as a model.	Templates	Guide Model Plan	Guía/Modelo Modèle Modelo	
Sides	The line segments that form a polygon.	The space next to something.	Side Siding Sided	Boundary Edge	Lados Côtés Lados	
Angles	Shape formed by two straight lines or surfaces that meet	a corner; a shape formed by two lines meeting at a point.	Angle Angled Angling Angular	Corner	*Ángulo *Angle *Ångulo	
Diagonals	A line segment that connects two vertices of a polygon that are not next to each other.	Joining two opposite corners.	Diagonally Diagonal	------	*Diagonal *Diagonal(e) *Diagonal	

I.   Morceaux & Pièces (French)

The French translation(s) listed for "pieces" in the word generation chart is "morceaux" and the cognate "pièces." Note that "morceaux" is the better translation for "pieces" in English despite its orthographic dissimilarity. "Pièces" in French is more frequently used to describe "items in a set" (e.g., my dinner set has 46 pieces), "works of art" (e.g., these pieces are from the impressionist era) and coins. (i.e., money).

II.   Properties

Before prompting students to contemplate and describe the "properties" of different two---dimensional shapes, it may be helpful to clarify the definition of properties. It is conceivable that students might confuse this meaning of properties with that related to one or a collective's possessions. Instead, make clear to students that, in the context of the math task, properties refer to the features or characteristics of shapes.

Dando vueltas a los Polígonos

La maestra de Milo le dío un cuadrado de papel para recortar a trozos y crear con ellos un rompecabezas. Despúes de probar distintas maneras cómo cortar el cuadrado en trozos, Milo le dijo a Sarah: "¡Mira, Sarah, he cortado el cuadrado en 8 pedazos y los he ordenado para formar un rectángulo sin ser un cuadrado también!"
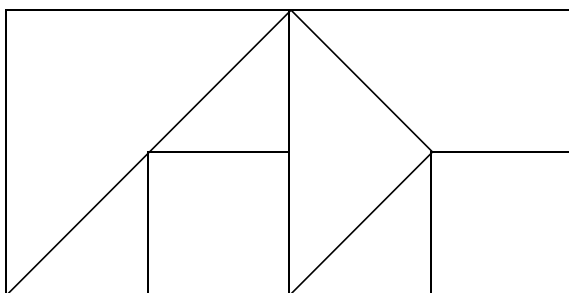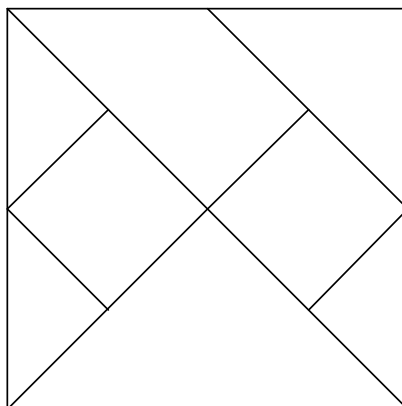
1. Sarah retó a Milo: "Muy bien, con las ocho piezas tu formastes un rectángulo exacto…"

…pero que pasará si aumento el rectángulo 1.5 de largo por 1.5 de alto…
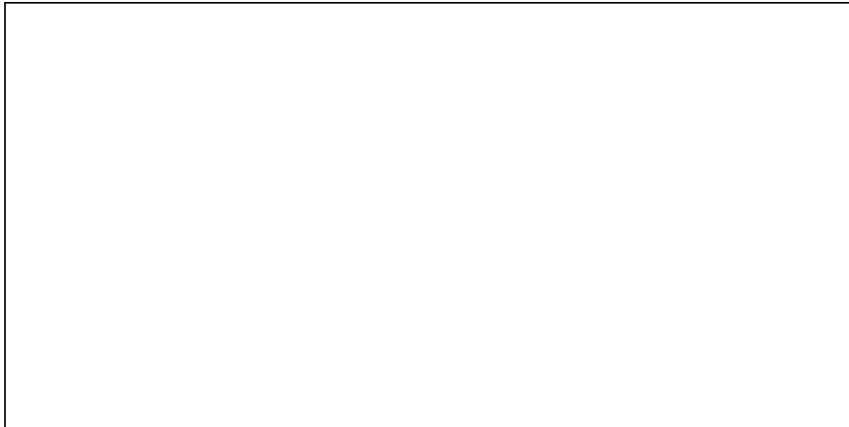
"¿Puedes descomponer este rectángulo en 8 piezas y ordenarlas para construir un cuadrado?"

a. Ayuda a Milo resolver la pregunta de Sarah, cortando este rectángulo en 8 piezas y ordenándolas hasta formar un cuadrado (hay una copia del rectángulo al final de este paquete). Debajo, pega o dibuja el cuadrado que formastes:

b. *Sin tomar medidas*,

   i. Explica la relación entre la logitud de un lado del cuadrado original de Milo y la longitud de un lado del cuadrado que hicistes.

   ii. Explica la relación entre el área del cuadrado original de Milo y el área del cuadrado que hicistes.

2. Ahora Milo quiere retarte a tí! Tu reto será recortar el cuadrado al final de este paquete en 6 piezas solamente, las cuales ordenarás para formar el rectángulo de abajo:

   a. ¿Puédes combinar las 8 piezas del cuadrado de Milo para formar un cuadrado de 6 piezas y formar un rectángulo que no séa un cuadrado? (encontrarás las 6 piezas en el cuadrado al final de este paquete).

   b. ¿Puédes recortar un cuadrado diferente en 6 piezas para formar el rectángulo siguiente?

c. Escoje una de las dos figuras de 6 piezas que hicistes en los apartados a) y b), y explica cómo el cuadrado se puede convertir en un rectángulo. Dibuja las piezas en el cuadrado y el rectángulo de abajo
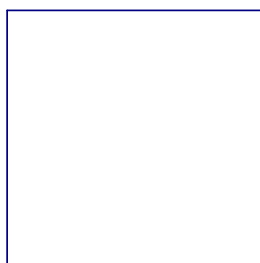
d. Para el mismo rompecabezas, describe cómo el cuadrado podría convertirse en un rectángulo.

e. ¿Que sabes sobre cada de tus piezas del rompecabezas que describistes en las partes c) y d)? Completa la tabla de abajo.

Dibujo de la pieza del rompecabezas	Nombre de esta figura	Propriedades de la figura

f. Sarah te dice, "Yo sé que todos los rectángulos tienen 4 ángulos rectos, 4 lados rectos, y que los lados opuestos tienen la misma longitud. Me parece que tu rectángulo final también tiene estas propriedades, pero necesito que tu me lo asegures." (Quizás quieres usar lo que tu sabes sobre las propriedades de tus formas (apartado c)) para asegurar a Sarah).

    i. Explícale a Sarah porque tu rectángulo tiene 4 ángulos rectos.

   ii. Explícale a Sarah porque tu rectángulo tiene 4 lados rectos.

iii. Explícale a Sarah porque los lados opuestos de tu rectángulo tienen la misma longitud.

Extensión:

Usa otro cuadrado al final de este paquete y crea un cuadrado con 8 piezas que, como el de Milo, pueda convertirse en un rectángulo. Las piezas de tu cuadrado no se pueden parecer a las piezas de Milo. Crea un rompecabezas en que las piezas esten simetricamente colocadas con respeto a la diagonal del cuadrado.
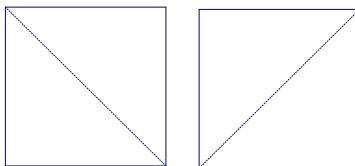
Aquí tienes un ejemplo de un cuadrado con dos piezas colocadas simetricamente con respeto a la diagonal.
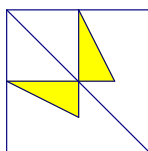
a) Dibuja el cuadrado que hicistes y enseña como lo dividistes en piezas que son simetricas con respeto a la diagonal. Después dibuja como colocastes las piezas para formar un rectángulo (que no séa cuadrado).

b) Nombra todos los ángulos de las piezas, sin medirlos.

c) Explicale a Sarah porque la forma final es un rectángulo.

Plantillas de las formas geométricas