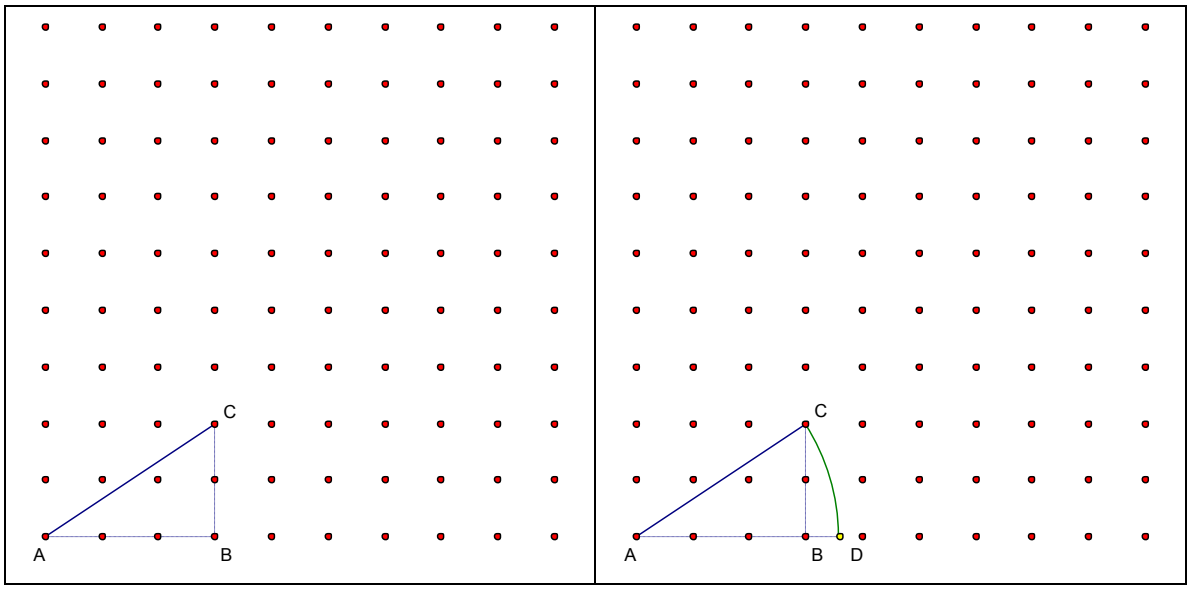
Going to Any Lengths

One Way to Estimate Lengths of Diagonal Segments

In the picture below, the length of segment $\overline{AB}$ is 3 units and the length of the segment $\overline{BC}$ is 2 units. What about the length of diagonal segment $\overline{AC}$? You can determine an estimate for the length of $\overline{AC}$ by holding a piece of string from A to C and, using A as a center, sweep out a circular arc until it intersects the line running through A and B. (This can also be done with a compass.) We have done that in the picture on the right. In that picture, D is the intersection point. From this, you can see that the length of the diagonal segment $\overline{AC}$ is between 3 and 4 units, and apparently slightly closer to 4 units.



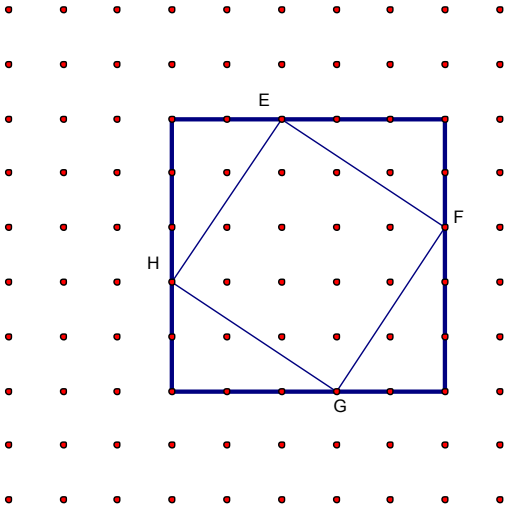
As you work on dot paper to locate diagonal line segments of specified lengths, there are two rules you must always follow:

- **Each line segment you locate must begin and end on a dot.**
- **You cannot measure using a ruler or other measuring device.**

Task 12:
Going to Any Lengths

1. Sketch a diagonal line segment on the dot paper shown below with
   a. Length between 2 and 3 units.
   b. Length between 4 and 5 units.
   c. Length between 5 and 6 units.
   d. Describe how you constructed these lines segments.

2. The picture below shows a quadrilateral EFGH, embedded in a square.



a. You say to your friend, Doubting Thomas: "Quadrilateral EFGH is also a square. It has 4 right angles and all of its sides are equal." Thomas says, "I doubt it." Explain how you would convince Thomas that EFGH has the properties of a square.

b. Calculate the exact area of square EFGH.

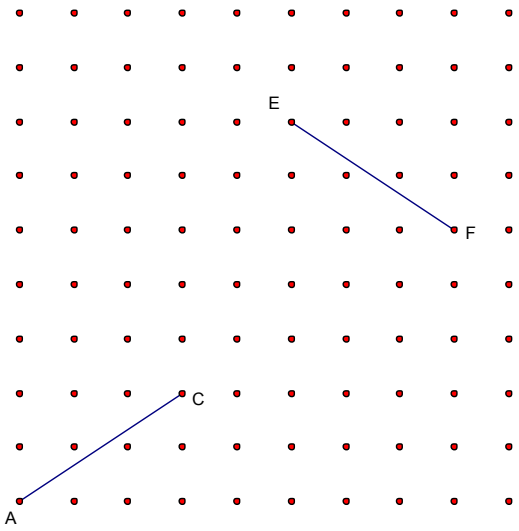
Explain how you calculated that area.

c. Use the area of square EFGH to calculate the exact length of segment $\overline{EF}$.

Explain why your method works.

3. You recognize that there are other line segments on your dot paper with the same exact length as segment $\overline{EF}$.

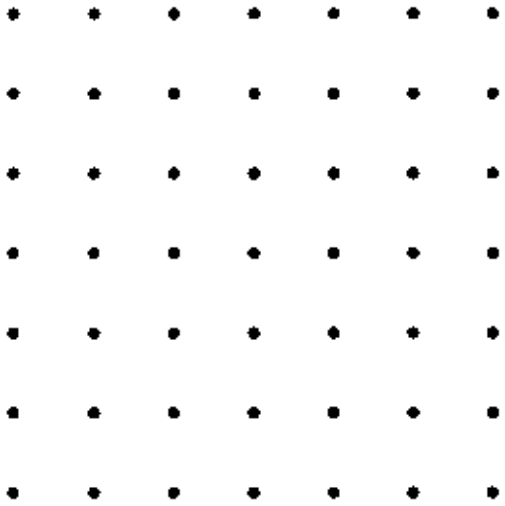
   a. You tell Doubting Thomas: "Segments $\overline{AC}$ and $\overline{EF}$ have the same exact length. Thomas says, "I doubt it." Explain how you would convince Thomas that you are correct (without using a ruler or compass to measure and without cutting one segment out to place it on top of the other).



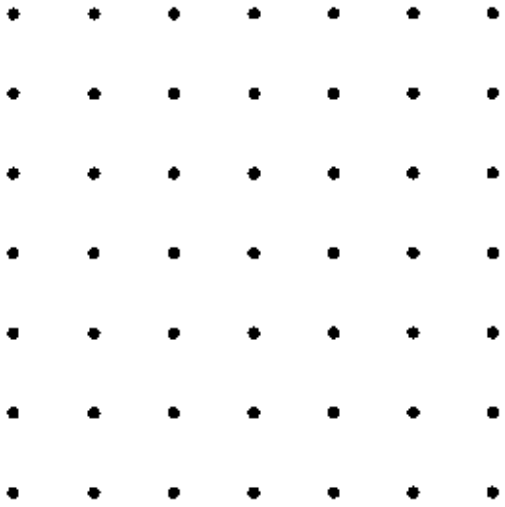
   b. Now sketch a line segment on the dot paper shown below that has length equal to $\sqrt{26}$ units. Explain how you calculated this length for the segment.

Warm-up for Going to Any Lengths

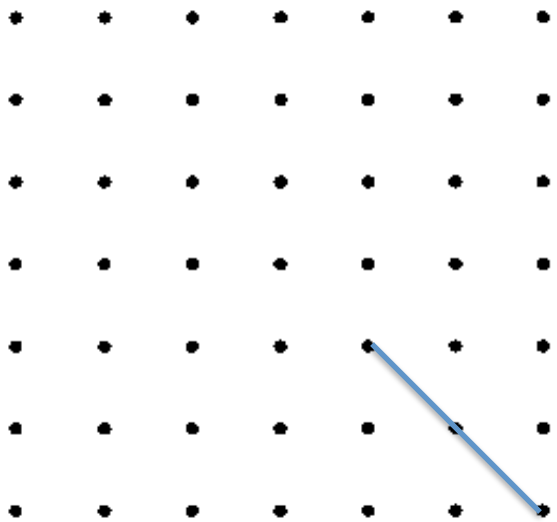
1. Sketch a line segment below with a length of 3 units.



Now sketch a line segment with a length of 5 units.



2. Imagine a boy in your math class tells you *"the line segment in the picture below has a length of 2 units."* Would you agree with him? Use string to measure the line segment in the picture below and explain your answer.



Potential Sentence Starters and Frames

Some ideas for sentence frames/starters that could be incorporated into your lesson are listed below. If you think a sentence frame/starter will be helpful, consider how will it support students' mathematical learning and/or development of academic language, and decide which sentence frame/starter (from the list below or that you create) would best support students' learning.

Starters

I notice that the diagonals from *problem 1* _____

_____

I know quadrilateral EFGH is a square because _____

_____

I calculated the area of quadrilateral EFGH by _____

_____

Frames

Quadrilateral EFGH is a _____ because _____
(shape)

I calculated the _____ of line segment EF by _____

_____

Academic Language

Students should have opportunities to see, hear, and write key mathematical ideas during this activity. There are some specific terms that students need to understand in order to engage in this task, and there are some additional terms and phrases that may surface as the students engage with the task. You may think of additional words or phrases that are key to this activity. As the task is introduced, solved by the students, and discussed, ensure that students have opportunities to experience (i.e., through discussion, pictures, and the use of gestures) and to build understanding for key words and phrases. Examples of words and phrases that may be involved in work on this problem include:

- Line Segment
- String
- Arc; Diagonal
- Unit
- Contstruct; Embed; Calbulate; Estimate
- Quadrilateral

Word Chart for Going to Any Lengths

Spanish, French, Portuguese
* = Cognate

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Line Segment	A straight line that links two points without extending beyond them.	Unbroken and bounded portion of line.	Segmented line Line segments Segmenting lines	Part of line Portion of line Section of line	*Segmento de línea *Segment *Segmento de linha	
String	---	Material twisted together to form a thin length.	Strings Strung Stringing	Cord Strand	Cuerda Ficelle Cordel	
Arc	Continuous portion of a curved line.	Something curved; curved structure.	Arcs Arced	Curve Portion of a circle or ellipse Path	*Arco *Arc *Arco	

Mathematical Thinking:
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Word Chart for Going to Any Lengths

Words and Phrases	Academic Language Meaning	Everyday Language Version	Other Forms of the Word or Phrase	Related Words or Phrases	Translation	Examples of word use with students
Unit	A single and whole quantity used for measuring or counting.	A single thing.	Units	Whole Single Quantity	*Unidad *Unité *Unidade	
Construct	To create new elements with previously measured or constructed elements.	To build; to assemble. To put something together.	Constructing Constructed Constructs	Build Make Create Form To put together	*Construir *Construire *Construir	
Embed	---	To fix or place firmly in a surrounded structure	Embedded Embeds Embedding	Implant To place within	Incrustar Incruster Embutir	
Quadrilateral	Four-sided polygon; four-sided two-dimensional shape.	---	Quadrilaterals	Parallelogram Rectangle Rhombus Square	*Cuadrilátero *Quadrilatéral *Quadrilátero	

Support Notes for Going to Any Lengths

I. "Line" vs. "Line Segment"

Students are asked in this task to locate and construct "line segments." It may helpful to clarify the difference and similarity between a "line" and a "line segment." That is, a line segment is an unbroken and bounded portion of a line; a line is infinite.

II. "Form" vs. "Construct"

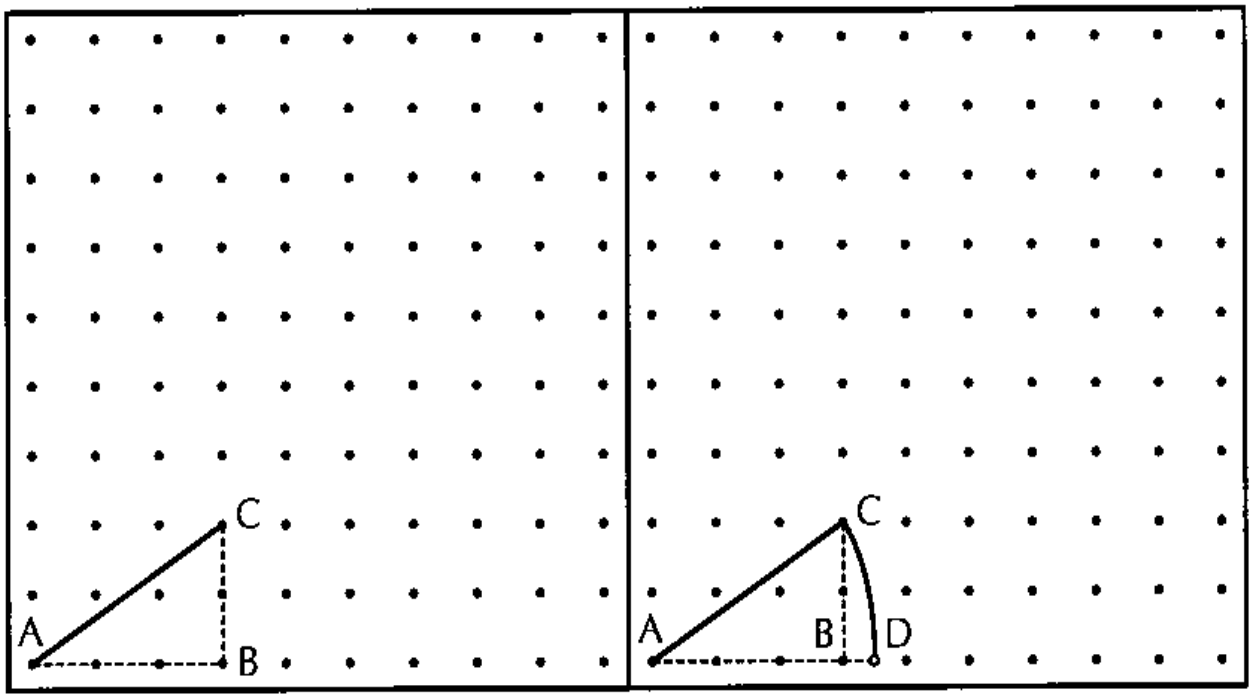
The teacher may wish to consider using the word "form" instead of "construct" as it appears throughout the problem (e.g., *Construct* a **square** with exactly **¼ the area** of the original square; *Construct* a **triangle** with exactly **¼ the area** of the original square). As in English, the word construct (i.e., construir, construire) evokes notions of erecting physical structures. "Form," on the other hand, may help more clearly convey to the student that s/he is to manipulate, change the appearance of the sheet of paper.

III. Work

"Work" is used in this task in the functional/operational sense (e.g., Bob's cell phone doesn't work). Some students may be more familiar with "work" as the verb describing an activity in which one engages to achieve a purpose or result, or "work" as the synonym for a "job" or "employment." You may find it helpful to address how "work" is used in the context of the task.

Yendo a Cualquieras Longitudes

En la imagen de abajo, el segmento de línea *AB* mide 3 unidades en longitud y el segmento de línea *BC* mide 2 unidades en longitud. ¿Cuánto mide el segmento de línea diagonal *AC*? Para calcular la longitud del segmento de línea *AC*, se puede sostener un pedazo de cuerda del punto A al punto C, usar el punto C como centro, y rodear la cuerda hacia el punto B hasta que se intersecte con la línea que recorre el segmento *AB*. (También se puede cumplir este paso usando una brújula). Este paso esta representado en el lado derecha de la imagen de abajo. En esta imagen, *D* es el punto de intersección. Se puede observar, a través de la imagen, que el segmento de línea diagonal *AC* mide entre 3 y 4 unidades en longitud, y aparentemente un poco más cerca a 4 unidades.



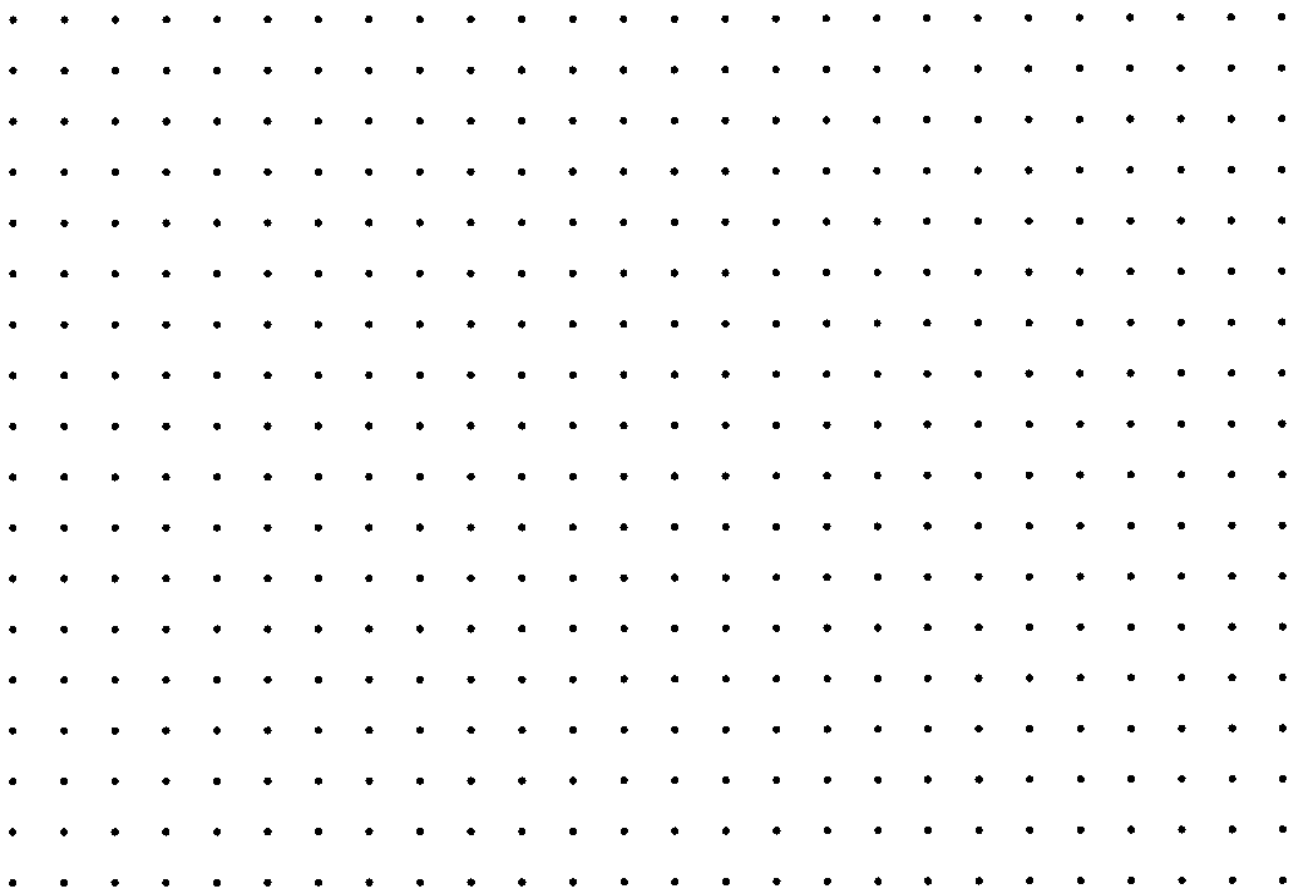
Mientras usted trabaje sobre el papel de puntitos para ubicar segmentos de línea de longitud especificadas, hay que siempre seguir dos reglas:

- Cada segmento de línea que usted ubica necesita comenzar y terminar sobre un puntito.
- No se puede medir usando una regla o otro aparato de medir.

Task 12:
Yendo a Cualquieras Longitudes

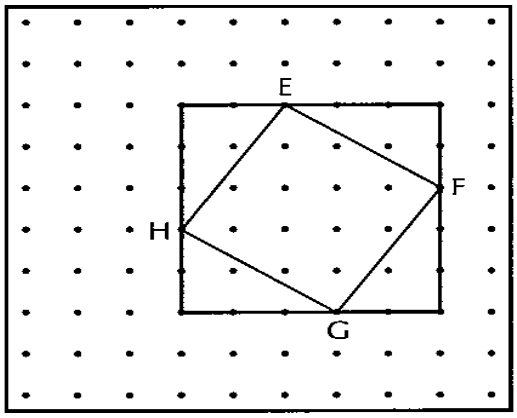
1. Dibuje un segmento de línea sobre el papel de puntitos de abajo con:

   a. Longitud de 2 a 3 unidades.
   b. Longitud de 4 a 5 unidades.
   c. Longitud de 5 a 6 unidades
   d. Describa como usted construyó estos segmentos de línea.



2. La imagen de abajo demuestra un cuadrilátero *EFGH* dentro de un cuadrado.

Task 12:
Yendo a Cualquieras Longitudes



a. Imagine que usted dijo lo siguiente a su amigo, *Tomás el Dudoso*,: "El cuadrilátero *EFGH* es también un cuadrado. Tiene cuatro ángulos rectos y cuatro lados equivalentes." Su amigo, *Tomás,* responde: "No lo creo." Explique como usted convencería a Tomás que el cuadrilátero *EFGH* tiene las propiedades de un cuadrado.

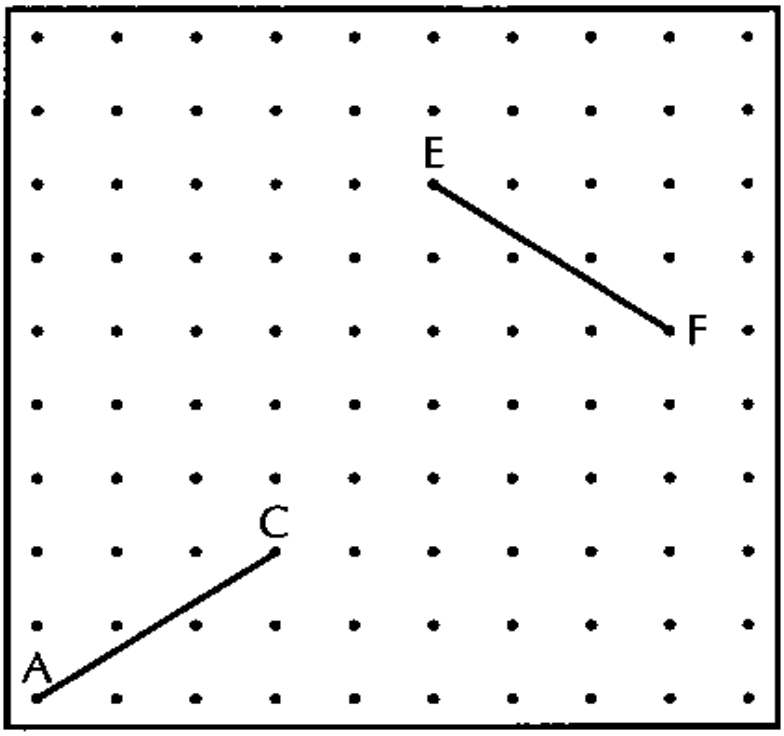
b. Calcule el área exacta del cuadrado *EFGH*. Explique como usted calculó el área.

c. Use el área del cuadrado *EFGH* para calcular la longitud exacta del segmento *EF.* Explique porque funciona su método.

Task 12:
Yendo a Cualquieras Longitudes

3. Usted nota que hay otros segmentos de línea sobre su papel de puntitos que tienen la misma longitud que el segmento *EF*.

   a. Usted dice lo siguiente a Tomás el Dudoso: "Los segmentos *AC* y *EF* tienen las mismas longitudes." Tomás responde "No lo creo." Explique como usted convencería a Tomás que usted está correcto (sin usar una regla o brújula para medir y sin cortar un segmento del papel para ponerlo encima del otro).



b. Ahora dibuja un segmento de línea sobre el papel de puntitos de abajo que tiene una longitud de √26 unidades. Explique como usted calculó la longitud de este segmento.